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THE MATHEMATICS TEACHER

Volume XLIII Number 7

Mathematics—Its Role Today*

By HEROLD C. HUNT
General Superintendent of Schools, Chicago, Illinois

LET me suggest at the outset as I bid you a delayed but no less sincere and hearty welcome to Chicago, where this week you have convened in annual session that the topic graciously assigned me, "Mathematics—Its Role Today," appeals to me tremendously. Surely it is most timely for today in the year 1950 we have come to the midpoint of the twentieth century—the midpoint which presents a strategic opportunity for both a backward and a forward look which permits an appraisal of what has been done thus far, what we are now accomplishing, and what trends our activities must take to make the last half of the century productive of the goals we seek.

By way of a backward glance over the past fifty years Professor Arthur M. Schlesinger, noted Harvard historian and distinguished Pulitzer prize-winner, has recently listed the ten historical events of 1900–1950 which have had the greatest effect in shaping the history of the world. May I remind you of Schlesinger's nominations for the half-century hall of fame? He suggests first the emergence of the United States as one of the two dominant world powers today as the most "world shaking" event of the first half of the twentieth century and then in chronological order lists World War I; the League of Nations; the political emancipation of women; the depression of the thirties; World War II; the practicability of atomic energy; the abandonment of colonial imperialism on the part of the major nations of the world; the formation of the United Nations; and finally the emergence of Russia as the second dominant world power.

This is an interesting and significant list, but I take issue with it chiefly because of its omissions. Lacking is any recognition, in adequate measure at least, of the tremendous upsurge of scientific invention and discovery so peculiarly characteristic of the past half-century which has witnessed the economic and social impact of the automobile, the telephone, the airplane, radio, television, the diesel engine, the wonder drugs of science—to list but a few.

Back in 1900, for example, there were but 10,000 automobiles and trucks in operation in the entire United States—today there are more than 41,000,000. At the beginning of the century the telephone was in its infancy—there were then but 1,350,000 phones as compared with more than 40,000,000 today. Radio was unknown in 1900—today, fifty years later there are more than 83,000,000 sets in use in the United States. Television, new in the communications area, has sold within the past three years almost 4,000,000 sets. During these past fifty years we have

* Presented before the Twenty-Eighth Annual Meeting of the National Council of Teachers of Mathematics, April 14, 1950.

become air minded and the pioneering efforts of the Wright brothers were translated last year alone in nine billion passenger miles flown by Americans!

Missing, too, in the Schlesinger list is any reference to the recognition of the importance of the individual, the supreme value and worth of human personality, the dignity of man. In my opinion, this is by far the outstanding characteristic of the past fifty years, finding reflection in both achievements and trends of significant interest and concern to the educator. Let me be more specific. In 1900 the work-week was well established at sixty hours; our national income was at 14 billion, 550 million dollars; our high school enrollment stood at six hundred thousand and in our colleges and universities two hundred and forty thousand young men and women were enrolled. Let's look at the picture in 1950! The work-week is now forty hours; our national income is in excess of two hundred forty billion dollars; high school enrollments last September were estimated at six and a half million while our colleges and universities enrolled approximately two and a half million. In fifty years while the population of the United States was but doubling itself—from seventy-five million to one hundred and fifty million—enrollments in our high schools and colleges were increasing ten fold. Elementary enrollments during the same period merely reflected population trends.

These facts I bring to you because no discussion of the role of any phase of our educational program today would be properly conceived without the realization of the conditions which brought us to our present position. It is well to recall then the soundness of the conviction that the vital issues of American education stem from the vital issues of our society and that the goals of democracy are but a reflection of the goals of that democracy.

And so in contemplating on the role of mathematics today we must take note first of its contribution to the democratic values in general education. We are all acquainted with the long list of these values to which mathematics contributes, including such processes, rights, and obligations as:

1. The settlement of conflicts by investigation and deliberation.
2. Justice before the law and the courts.
3. The right to work usefully and creatively.
4. The right to security with freedom from old age, worries about sickness, and unemployment—personal security.
5. The right to fair pay and adequate working conditions.
6. The right to adequate food, clothing, shelter, and medicine.
7. The faith in education as a means of personal and social improvement.
8. The right to rest and recreation; the right to own property, provided, however, that it is controlled in the interest of the common good.
9. The highest development of the individual is the aim of the social system; and the common good is to be preferred to that of special groups.

In order to fulfill its designated role and to make its most effective contribution, mathematics must be functional. Mathematics is the oldest of all sciences and throughout its long history a sturdy framework has been developed, a framework which includes arithmetic, geometry, algebra, and trigonometry. This structure cannot be rediscovered by the individual pupil or school solely on the basis of individual needs and interests. To make mathematics functional, therefore, there must be an initial period of systematic study, within a desired range, of the underlying mathematical concepts, principles, skills, and modes of thinking.

It is fully realized, however, that mathematics grew out of life situations and that the meeting of everyday problems has brought about an interaction of theory and practice that has led to further development. To make the most of the science there must be proper emphasis on the interrelations between mathematical theory and its many-sided applications. Functional competence in mathematics may be described as an outgrowth of a continuous and painstaking emphasis on the categories of understanding, mastery, and transfer.

In the primary grades mathematics is

effectively related to the pupil's everyday experience. Girls and boys bring to the classroom a wealth of number experiences, acquired in work and play. The social setting for such experiences as the making of purchases at the store and reading numbers on houses, on automobile licenses, on price tags, serves to relate the meaning of mathematics to individual and personal activities and to activities involving group participation. Throughout the elementary school years the personal interests and experiences of the pupil provides this social setting to the mathematics program. Both the home life and school life of the pupil place him in frequent contact with problems requiring the application of mathematical skill. If the program keeps pace with the child's ever-growing needs, it is serving its functional purpose. Use of social problems in the teaching of mathematics in the elementary school assists the pupil in the mastery of the essential elements through the motivation of formal work which these experiences provide.

During the high school years pupils develop an increasing interest in adult activities. They prepare themselves for participation in adult economic and social affairs through their growing awareness to problems of buying and selling, wages, budgets, insurance, investments, taxes, borrowing money, paying interest, and the like.

The problem of teaching high school mathematics for social change is, however, not at all simple. The present high school mathematics curriculum, growing as it did through inheritance from institutions of higher learning, is still cloaked in the language of the specialist and retains the point of view of the disciplinarian. A difficult area at best, mathematics has long been a battleground for reform in content and method.

The changing high school population, which since 1900 has become steadily less selective, has had, of course, much to do with the development of the newer pattern. As enrollments have increased, a larger proportion of the students either have been uninterested in studying algebra and geometry or have found these subjects too difficult. In 1900 the number taking mathematics courses was five times the number not taking them. Today this situation is almost completely reversed, although by adopting the point of view of the psychologist and finding ways of motivating the work, teachers are becoming more successful in attracting students to mathematics courses and convincing them of the significance of the study and its importance for the development of their own competence.

The situation today therefore, as I see it, is this—the high school population is divided into two groups, the smaller of which includes those pupils who study mathematics because of interest or felt need, and the larger of which includes those willing to get along without mathematics because they dislike the study or find it too difficult. Our big problem today is to develop a mathematics curriculum that will attract and will be profitable for this latter group. Planning of the courses around social units, which make the mathematical work seem incidental and which provide for learning through use, will likely lead to success in securing the interest of increasing numbers of high school students.

About three years ago a committee of the American Mathematical Society announced its conclusion that something is very wrong with the teaching of mathematics in American schools. I quote from a report which appeared in the *Philadelphia Evening Bulletin* of September 6, 1947, which states that:

> The committee thinks for one thing that the study of statistics should be made a part of a liberal education. Statistics do, indeed, turn up everywhere in the American way of life. Batting averages, betting odds and stock market quotations are statistics. Safety authorities predict the casualties of a holiday week-end by studying statistics. The popularity of radio programs, the chances of a political candidate and the probable weather for a week from Wednesday are figured by statistics. The average American is surrounded by figures almost from the cradle to the grave. It has become a minor mathematical

problem to determine the cheapest way to travel. The wage earner's pay is a product of mathematics, especially subtraction.

These practical applications and others which readily come to mind may be utilized effectively both to attract the larger group and to provide greater competency in democratic living.

A unit on the budget of high school students, for example, will contribute to the understanding of the democratic values of the right to adequate food, clothing, shelter, and security. To carry out the project students may provide a report of the income and expenditures for which they were responsible during the previous week. From this information individual budgets may be made for the next several weeks with emphasis on the fact that each student is to live within the limits of this budget estimate.

The activity may be extended and made to have greater meaning by ascertaining the per cent of savings and the per cent of each expenditure account. A comparison of the budgets of various types of people—laborers and professional men for example—affords additional insight and understanding.

Graphs may be made of expenditures as budgeted and as expended. Savings-income graphs will portray dramatically the requirements for the various income groups. Life insurance as a means of saving may be included. A study of living costs serves to strengthen the student's feeling of the importance of budgeting to his personal security.

Such a unit is certain to demonstrate the importance of mathematics in the life of the student as he approaches and assumes adult responsibilities.

Units on home building and upkeep likewise contribute to the appreciation of democratic values. In such a unit geometry may be applied to the locating and designing of the building. Greater appreciation of the right to food, clothing, shelter, and security is assuredly developed through the study of purchasing activities of the home, insurance savings account, and the apportioning of family income.

Units on how the student spends the twenty-four hours of the day not only develop mathematical skill but provide a concept of work as it has contributed to the improvement of social and economic conditions. Units on wages and salaries impart a clearer recognition of the right to work usefully and creatively, the right to fair pay and adequate working conditions, and the highest development of the individual as the aim of the social system.

Units on safety permit the development of further mathematical skill and afford increased awareness to individual responsibility in protecting life and property. The problem of the determination of the distance necessary to stop an automobile is but one example of the mathematical applications and social implications of the area.

Many other examples could be cited. I am certain that mathematics teachers are well aware of the possibilities. It remains, however, that there are two major functions which you as teachers of mathematics are expected to perform (1) to lead the pupil to a realization of the objectives of mathematics and (2) to contribute to the objectives of general education. In meeting the second obligation, mathematics cannot remain static. The curriculum must be adapted to the changes that are taking place in social and economic life. When materials become obsolete, they must be discarded, and socially useful subject matter must take their place.

What then is the picture which educators are painting of the significance of mathematics? Whether true or false, it determines the actual status of mathematics in our schools and underlies the issue of functional competence. Certain critical appraisals and demands which have recurred more or less regularly are:

1. The mathematical curriculum, "for all but the few," should stress only those minimum essentials actually needed in everyday life.
2. All the mathematics taught in the school

should be derived from actual life situations.

3. Academic mathematics of the usual type does not "meet the needs" of "the other 85 per cent" and should be reserved "for the few."
4. "All but a few" can get along without any mathematics in the secondary schools.

The seeming determination to protect "all but the few" against learning anything beyond the minimum essentials gravely endangers a democracy like ours. We must remember that "there is no surer key to unlock all sorts of doors than mathematics." We have no right to close these doors to "all but the few."

Through mathematics in general education, experience of formulation and solution, data, approximation, function of several concepts, and other processes, the democratic values will be reconstructed so that they will definitely stand for things more valuable than the "isms" could ever sponsor and by application they will be extended to a larger and larger number of people.

And so the role of mathematics today is that of a pillar of our democratic framework. Mathematics provides the mainstay for our democratic concept for the development of competence for each individual. Achievement in mathematics affords rewarding experiences to all and a satisfying feeling of effectiveness in solving individual problems. The confidence which is thus imparted contributes to the objective of self-realization which is readily accepted as one of the four major purposes of education in a democracy today. To the other three of these well-known objectives does mathematics likewise contribute. Good human relations are more firmly established through problem-solving activities which involve cooperation, participation, and sharing. We have pointed out in great detail the ways in which mathematics fulfills the purpose of economic efficiency. And in discussing the democratic values in general education to which mathematics contributes we have dwelt long and forcefully on the achievement of civic responsibility.

At the midpoint of the century, then, as we plan for the fulfilment of our responsibilities in the next fifty years, it seems certain that the role mathematics will play, as we have detailed it, will satisfy the criteria for equipping the school pupil of the second half of the century to take his place as a member of a family, as a producer, as a consumer, as a citizen, as a taxpayer, as a voter, and as a member of society at large.

In playing this role today and in the years ahead mathematics will assume an increasingly important part as it continues to adapt itself to the interests of those who study it and to the needs of the times. Toward the development of the competency of the individual mathematics contributes now and will continue to contribute most significantly. For the realization of this important objective does mathematics perform its most purposeful service, in the fulfilment of this goal does it play its stellar role.

In the next fifty years—with, as Dr. Gerald Wendt, distinguished American scientist, predicts, a twenty-four hour work week, a life span of from 85 to 90 years (as contrasted with 62 to 67 years today) and an average annual income of $12,000 (as contrasted with $3,000 today and $1,000 in 1900)—in such an age of science and specialization, of atomic and hydrogen power put to peaceful uses, in such an age will mathematics move still further forward in adapting its processes and procedures toward the development of still greater individual competence. The challenge is then—to make our field functional, at all times adapted to evidenced needs, capable of fulfilling requirements however novel and revolutionary they may seem. As mathematics teachers we can accept this challenge wholeheartedly, confident that the serviceability and the versatility of our subject will meet the most exacting as well as the most changeable requirements. We have within our grasp, in our very hands, a tremendous force for good. Let us apply it at fullest capacity.

A Job Survey as Class Motivation in General Mathematics

By John R. Eales
Secondary School Coordinator, Los Angeles County Schools, Los Angeles, California

In what phases of mathematics should a junior high school student be proficient to be adequately qualified to compete in economic employment? All teachers could give a general answer to that question. Few teachers could be highly specific in answering it. Miss Leone McGowan and Miss Florence Treadway of the Franklin D. Roosevelt High School in Compton, California did not try to answer it. They led 152 students in four general mathematics classes to find the answer for themselves through a job survey.

The survey was conducted as follows. First, teachers and students discussed the problem in class. Second, class surveys were made on job interests, personal qualifications, employment opportunities available, knowledge of various types of mathematics and mathematics applied on the jobs. Third, committees discussed the surveys. Fourth, elected delegates interviewed business employers. Fifth, these delegates carried on panel discussions with the classes. Last, this same panel presented its findings before the mathematics teachers.

As is too often the case, general mathematics did not seem to challenge many of the students as it should. They were taking the course but they seemed to have no particular interest in it. They could see little value in the problems contained in their textbooks. Hence, as a means of stimulating interest the students were encouraged to take some class time to evaluate their progress in the year's work and to clarify their purpose for taking the subject.

During this evaluation period the classes all became interested in business needs for mathematics and in the immediate requirements in mathematics for employment. Is mathematics used much in business after you get out of school? Does a young person need to know much mathematics in order to get a job? If so, do some occupations call for the use of more mathematics than do others? These, and other questions of the same nature, were asked in all the classes.

It seemed that employment was a major interest in the lives of these students. They were all eager to talk about jobs they held, had held, or hoped to hold in the future. These students were only in the ninth grade but already the thought of present employment and of preparing to enter a chosen vocation in the future loomed large in their thinking.

Several class periods were given over to discussing mathematics as it applied to their respective jobs and also to discussing the type of training they were getting in their present mathematics course. As a result of these discussions, it was decided to make a survey in the four general mathematics classes regarding jobs now held by the students, job interests of the students, the occupation of the fathers of the students, and other pertinent information.

Each of the four sections then chose two delegates to serve on a committee to compile the information gained from the survey and to report the results to the classes. A summary of the report made to the classes follows:

I. A survey on occupations of fathers shows that the 20 leading positions or jobs of fathers were in order of scores given:

1. Building contractor
2. Salesman
3. Machinist
4. Carpenter

5. Painter
6. Welder
7. Telephone worker
8. Fireman
9. Oil Worker
10. Steel worker
11. Produce business
12. Armed services
13. Realtor
14. Grocer
15. Butcher
16. Plumber
17. Service station worker
18. Radio shop and repair
19. School teacher or principal
20. Janitor

II. Total number of students who intend to follow the father's trade or occupation—20.

III. Total number of students who now have jobs or will have them in the summer—34.

IV. The different kinds of jobs students now have, in order of their scores:
1. Baby sitting
2. Paper delivery
3. Lawn mowing
4. Housework
5. Repairing scooters and bicycles
6. Grocery store helper
7. Bakery helper
8. Carpenter helper
9. Gas station helper
10. Repairing radios and electric appliances
11. Cook's helper
12. Musician

V. Jobs preferred by girls for a period of the next three years, in order of their scores:
1. Department store clerk
2. Office assistant
3. Baby sitter
4. Waitress
5. Stock girl
6. Usherette
7. Nurses' aide
8. Music store clerk
9. Florist
10. Telephone operator

VI. Jobs preferred by boys for a period of the next three years, in order of their scores:
1. Mechanic
2. Grocery store clerk
3. Baseball player
4. Office assistant
5. Carpenter
6. Paper carrier
7. Gas station helper
8. Market helper
9. Waiter and dishwasher
10. Fruit picker
11. Cabin boy
12. Farmer

VII. Job requirements expected of students when employed by the public:
1. Work permits
2. Social security cards
3. Birth certificates
4. Age limits
5. Previous experience
6. College training
7. High school diploma
8. Health certificates
9. Training or knowledge of specific job

VIII. Mathematics used on jobs now held by students:
1. Making change
2. Cashing and writing checks
3. Operating cash registers
4. Reading scales and computing cost of articles
5. Weighing cartons and boxes
6. Measuring baby formulas and cooking recipes
7. Checking miles per gallon of gasoline
8. Measuring floor surfaces in laying linoleum
9. Computing hours and time on clock
10. Computing wages when paid on a time basis
11. Adding golf scores
12. Checking water temperatures in swimming pool

By the time the results of this survey were made public in the classes and had been discussed, the interest had grown so great that the students suggested that an additional step be taken. They desired to interview some employers so as to get first hand knowledge of their attitudes and opinions regarding inexperienced workers and their need for a strong mathematical background.

The elected delegates of the four classes selected five business firms to visit. The four girl delegates went to the Security First National Bank in Lynwood and to the Telephone Company offices in Compton. The four boys had interviews at the Powell Manufacturing Company, Curries Ice Cream Company, and a gasoline station.

All these visits were carried on during the lunch period. Miss Treadway accompanied all the groups.

After the interviews had been completed, the delegates came together, compiled their findings, and prepared a symposium presentation for the classes. This presentation was received with a great deal of enthusiasm by the students and lively discussion resulted from the reports.

Mrs. Florence Farrand, Chairman of

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Mrs. Florence Farrand, Chairman of

the Mathematics Department, sat in with one of the classes as the report was being given and was much interested in it. She suggested that the symposium be repeated for the benefit of the teachers. Miss Blanche Taylor, Principal of Roosevelt High School, who had supported the job survey project from its inception, also felt that teachers would profit by hearing the presentation. As a result the students gave their information to a staff meeting.

The information presented by the symposium to the classes and to the staff was quite general in nature. Only in a rough way did these freshmen ascertain the specific phases of mathematics in which a junior high school student must be proficient to be adequately qualified to compete in economic employment. The result was, of course, in accord with the teachers' original expectations.

Both Miss Treadway and Miss McGowan considered the job survey to be an outstanding success. The students did learn from the survey that mathematics is a definite factor in their vocational choice. They learned that different types of employment require different amounts and kinds of mathematics. As a result, the importance of the subject was brought to their attention.

The teachers felt that the chief value of the project lay in its motivating power. During the remainder of the school year, the classes all showed greater interest in their work in general mathematics. The students' achievement in the subject was much improved. In addition, some capable students who had planned to discontinue the study of mathematics at the close of their present course were stimulated to plan for further study in the field.

This writer has undertaken to report on this project because it is felt by him that these teachers have made a real contribution. Class motivation has long been a problem confronting mathematics teachers. In carrying on this job survey Miss Treadway and Miss McGowan have demonstrated that the vocational interests of students of junior high school age can be used as a basis for stimulating interest in the study of mathematics.

Such motivating activities have been criticized by some teachers because they take up time which otherwise might be devoted to the teaching of subject matter. It is true that projects of this type take time. But is it time wasted? This writer feels that it is not.

Our students of psychology have learned much about how learning takes place. They point out that motivation is basic to successful learning. In theory, then, a project aimed at motivation should not be a waste of time. In addition, in practice, this motivating job survey was not a waste of time. Both teachers involved in it report that before the project, the subject did not seem to challenge the classes, while after the survey, all classes showed a greater interest in the work. In addition their achievement in the subject was much improved.

It would seem that all teachers of mathematics might profit by spending more time than they now do in "selling" the subject to their classes. Helping students to appreciate the practical value of mathematics is not a waste of time. It is an essential part of the work of every teacher.

Mathematics an Essential of Culture*

By Walter H. Carnahan
Purdue University, Lafayette, Indiana

About 500 B. C. Phidias carved some statues, and ever since that time men of culture have been expected to know something about him. Four hundred fifty years ago Raphael painted some pictures, and it is now an accepted mark of culture to know a little about him. About the time the United States had its painful national birth, Mozart composed music, and to admit complete ignorance of Mozart today would be culturally humiliating.

Man is the only creature on earth that has ideals of beauty, power, and goodness and tries to give these material form as a means of conveying them to other men. We admire, honor, and envy those who have the ability to give objective expression to these ideals through the media of the various arts. But these ideals are man-conceived and are restricted by the limitations of those who try to express them. Man knows that his music, his paintings, his poetry, his sculpture, his architecture are of his creation even though the spiritual ideal that inspires them may be outside and beyond himself. Being man-made, his art has all the limitations of its creator. Man is eternally trying to find reality and truth that transcends the power of his art to express, something more enduring than a sunset or the canvas and the paint by which he tries to preserve it, something more fundamental than the harmony of sounds or the vibrations of strings or reeds, something more universal than the comedy or tragedy of the lives of men and the dramas by which they are portrayed on the stage. He tries to understand how the universe is put together, how it behaves, what sets and keeps it in motion, what it has been and what it is to become. And so a great compulsion drives him to an attempt to understand natural law, and ultimately he is forced to invent a language in which he can express nature's laws. He is driven to mathematics. It is often the great artist who most clearly realizes the limitations of his art and turns to mathematics as a means of deeper penetration and more powerful revelation of the ultimate abstract ideal which is natural law.

It should be an essential part of general culture to know something of the steps by which mankind has come to some small understanding of the laws of the universe in which he lives and the language in which those laws are discovered, demonstrated, and expressed. In other words, some acquaintance with mathematics should be considered as essential to general culture just as is an acquaintance with literature, music, or painting.

This paper has a two-fold purpose: to give some examples of well-known persons (at least well-known in their own day) who turned to mathematics in order to achieve a deeper penetration of truth and beauty than could be gained by single-minded devotion to the art for which they were most widely known, and to make a small contribution to the dissemination of general culture as it relates to mathematics. This will be done by recounting certain facts from the biographies of selected persons.

Christopher Wren

On a tablet in the crypt of St. Paul's Cathedral in London, the visitor reads this inscription: "If you seek his monument, look around you." In the tomb beneath lies Christopher Wren, the man whose dream has been fashioned into the stone of the building. Look around you, look above, look below. Go outside and look at the double rows of columns, the majesty

* This article follows logically another of the same title and spirit which appeared in The Mathematics Teacher for April 1948.

of the great dome, the perfect proportions, the symmetry. Visit other great churches and public buildings in London and other English cities and behold other monuments to this renowned artist, enduring monuments of his own designing.

In 1666 one of the greatest fires in history destroyed the city of London. Thousands of homes and shops, scores of public buildings, and dozens of churches were reduced to ashes. When the fury had passed and the smoke had lifted, plans for rebuilding were evolved. An over-all city plan was needed, and designs and specifications for public buildings and churches were called for. City officials, bishops, and the King of England sought the most capable architectural service the realm possessed. In this circumstance, all eyes turned to one man, Christopher Wren. So well did he dream and plan and execute that three centuries of time have become three hundred years of testimony to his architectural greatness.

The architectural achievements of Christopher Wren are space-filling and sense-appealing. Small wonder, then, that few laymen know him as the great mathematician that he was. Among other contributions he discovered the method for finding the length of any arc of a cycloid, and determined its center of gravity. The cycloid has been called "The Helen of Geometry" because of the beauty of the curve and because of the mathematical wars which it has inspired. To have made two outstanding contributions to the glories of Helen are more than one man's share of mathematical honors. A friend of Isaac Newton, he was in necessary league with other friends of that great mathematician to prod him gently but constantly to make known his discoveries. Wren was a great architect by compulsion of circumstance, but there can be little doubt that at heart he was a mathematician, finding in mathematics a certain beauty, power, and permanence which he could not realize in the arches, domes, and columns of St. Paul's.

Geoffrey Chaucer

Six hundred years ago, England had a citizen who was always at her beck and call for any odd jobs of state. He was a chore and errand boy who could be entrusted with all kinds of minor responsibilities which must be done faithfully. He would work at any assignment at home or abroad. Today he would be in London Town, tomorrow at Cambridge, next week in France, next month in Holland. Now he would be negotiating a minor trade agreement; at another time he would be checking timber purchased for His Majesty's navy. He was supervisor of this forest, guardian of that bridge, collector of this revenue, dispenser of that fund, recipient of this stipend, that annuity. Like Martha, sister of Mary, he was troubled about many things. Sometimes his salaries and pensions promised to make him rich; then his circumstances would change and he was suddenly in want.

In the midst of all these activities, he found time to be a prolific writer. He translated literature from other languages into English; he wrote original tales, narrative poetry, and lyrics. Once when his finances were in an especially bad way, he wrote a *Complaint To My Purse* as a hint to the king to deal more generously with his servant, which appeal the king heeded.

This was Geoffrey Chaucer, great contributor of some of the most delightful poems in early English literature. Amid all his busy-ness he found time to write the *Canterbury Tales* which began:

Whan that Aprille with his shoures soote
The droghte of Marche hath perced the roote
And bathed every veyne in swich licour,
Of which vertu engendred is the flour;
Whan Zephirus eek with his swete breeth
Inspired hath in every holt and heeth
The tendre croppes, and the yonge sonne
Hath in the Ram his halfe cours y-ronne,
And smale fowles maken melodye,
That slepen al the night with open ye,
(So priketh hem nature in hir corages)
Than longen folk to goon on pilgrimages,
And palmers for to seken straunge strondes,
To ferne halwes, couthe in sondry londes;
And specially, from every shires ende
Of Engelond, to Caunterbury they wende,
The holy blisful martir for to seke,
Than hem hath holpen whan that they were seke

When he was fifty years old, Chaucer felt himself old, weary, and disillusioned before his time. Perhaps he needed a long vacation; perhaps he needed another pilgrimage to Thomas à Becket's Tomb at Canterbury; perhaps he needed contact with something deeper than poetry, more permanent than a king's favors, more secure than international treaties. He turned to mathematics. To be sure, he did it half apologetically as a man goes to the circus so that his children may see the elephants, but he did devote himself to the subject quite wholeheartedly for a while. He wrote a book on the astrolabe "for litell Lowis My Son." Do not be deceived, it was written by Geoffrey Chaucer for Geoffrey Chaucer.

Better books on applied mathematics have been written but none that bears more eloquent testimony to the power of mathematics to contribute to the spiritual necessities of man. A busy, harried man, a poet with a constant stream of beautiful ideas running through his mind, took time out to make contact with one of the permanent sources of man's strength, and wrote on mathematics.

Galileo

One day in the year 1585, word spread among the faculty of the University of Pisa in Italy that a young man of the city, twenty-one years old, had discovered a new and extremely useful and simple scientific principle. The principle was that of the hydrostatic balance. The young man was Galileo Galelei, son of an impoverished nobleman. Letters and traveling scholars carried the news all over Europe and Galileo was widely and immediately famous. The Marchese del Monte of Pesaro sent for him and offered all the financial support and social sponsorship which the young man might need. Three years later Galileo wrote a book on the center of gravity of solids which further directed the eyes of the scientific world to him as a great leader. There followed two years of remarkable experiments and demonstrations culminating in the famous gravity demonstration at the Tower of Pisa, with its proof that what the great Aristotle had taught, and the world for two thousand years had accepted, was an important scientific error.

In 1609 Galileo began the construction of telescopes which he used to reveal other scientific truth and to show men the grossness of certain prevalent scientific errors. Theologians discovered that their doctrines and Galileo's science were not in agreement. He was warned to make his science conform to their religion. How can one make science conform to the requirements of any man's command? Galileo went on with his researches. In 1623 he was cited to The Inquisition at Rome where he was tried under threat of the torture chamber, found guilty, and sentenced to spend the remainder of his life in retirement.

What are the thoughts of a scientist who must live in seclusion the last ten years of his life under the sternest requirement to say nothing, write nothing, think nothing of which entrenched authority does not approve? Perhaps it is his opportunity to evaluate all truth which he has known, that which others have found and passed on, and that which he himself has discovered and revealed. In this review he will find some tenets that are contestable and some that are undebatable. He may well take pride in having contended for truth which others denied, but he must have a greater feeling of confidence and security and mental peace in that which no one has found it possible to challenge.

During the last ten years of his life it may well be that Galileo often recalled that day in his seventeenth year when he accidentally overheard a teacher giving a lesson in mathematics. The inherent power and beauty of the subject struck him so forcefully that he went at once to his father and asked to be allowed to drop his medical studies and take up mathematics. From that day on he had devoted much time to the subject. He had made some wonderful discoveries which had brought great satisfaction and never a

moment of conflict or humiliation.

Why had no tribunal summoned Galileo and commanded him to cease teaching that a thrown ball follows a parabola, or that a cycloid is stronger in many structures than a circle? Why had no authority ever challenged his assertion that "Nature's great book is written in mathematical symbols"? The answer lies in the nature of mathematics itself, its firm resistance to every attack, its strong endurance, the eternal life which it has to take root and to sustain itself and to grow. Let us hope that in his last harried years Galileo realized to the full the very real spiritual benefits which mathematics can confer.

Christian Huygens

In the year 1655 the stout Dutch burghers of The Hague filled the streets with the murmur of a great piece of news concerning a discovery made by one of their young fellow townsmen. "They say he has invented a wonderful new way of grinding lenses." "They say new life will be put into our industry of lens making, and The Hague will become world leader." "It is said the lenses produced by the new method are so perfect that it is possible to make telescopes with which one can see the satellites of Saturn and can study the details of the nebula in Orion." "Just imagine the son of our old friend in the next street has done this great thing, young Chris Huygens."

That was the sensation of the year 1655 in The Hague. The next year there was another. "Have you heard? Chris Huygens has made another great invention; this time it is a new kind of clock, a more perfect one than the world has ever seen; no more burning of varicolored candles to tell time; no more measuring time by the dropping of water; no more depending on sundials. Chris Huygens has made a clock as accurate as the stars by causing it to be regulated by a vibrating pendulum. This will bring a great new industry for the city; prosperity and wealth such as we have never known. I tell you, the city may be justly proud of this citizen."

These were not the last of the occasions when the citizens of The Hague became excited about the achievements of Christian Huygens. In fact the whole of Europe soon heard about his achievements. The king of France heard about the young scientist and inventor and conceived the idea of taking him to Paris as an adornment of that city. He held out the bait of a liberal life pension, and Huygens went to make his home in the French capital for fifteen years. That was all right, because such a genius as his soon outgrows the boundaries of any state, and the accident of postal address means little.

How shall a man spend his time when he has acquired world fame and independent wealth before he is thirty years old? He can buy a string of race horses; he can gamble at Monte Carlo; he can eat, drink, and be merry; he can travel, live at ease, and bore himself to death.

Or he can do as Christian Huygens did. Huygens knew that the mathematicians of his time were bringing in such a day of power and glory in the subject as the world had never known. As a scientist he had learned to use mathematics as a powerful and indispensable tool. He knew that the mathematicians always have more than their share of pleasure. Furthermore he knew that people of real culture must know something of mathematics just as they know about music, art, and drama. Consequently he devoted himself to mathematics so effectively that he lives in the pages of the history of that subject as one of the great original contributors. Furthermore he was the inspirer and encourager of Leibnitz and thus made an indirect contribution to the development of mathematics.

Huygens invented the theory of evolutes and proved that the cycloid is its own evolute. He was one of those who proved that the tautochrone is the cycloid, and made use of this fact and his knowledge of evolutes to design the cycloidal pendulum

clock. He made contributions to the mathematical theory of probability.

Mathematics has lost none of its power to catch and hold the interest of men which it exerted over Huygens. It requires only to be presented in a favorable light for the consideration of intelligent persons.

Pierre de Fermat

Beginning about the year 1635, the citizens of the French city of Toulouse became accustomed to a reserved man who each morning walked unhurriedly and quietly along the streets to the parliament building, up the steps, and through the tall doors. At the end of the day, he could be seen regularly descending the steps and following the same streets to his modest home. "Who is he?" one would say. "He is the new councilor," another would answer, "one more to consume our substance and take bribes for giving counsel favorable to those who rule over us."

But as years passed if a stranger in Toulouse should ask a citizen, any citizen, who the dignified man might be, he would get a quick smile and an enthusiastic reply. "Ah, that is our councilor, a just, honest, and fearless man, a good citizen, a hard worker, Pierre de Fermat. Every morning up the streets and up the steps to the parliament and to his duties at the same hour; every evening down the streets and up the steps to his home at the same minute. And when his house door closes behind him—what? Who knows? He does not come out again to eat, or drink, or talk with others. Perhaps he dozes by the fire; perhaps he drinks himself drunk. Who can tell? Who cares so long as he is just and honest?"

Fermat's long evenings, holidays, and Sundays were his own to employ as he chose, either to drink, or to sleep by the fire, or to visit the gaming rooms, or to talk politics, to practice alchemy, or to dress and seek bright lights and gay companions. He did none of these things. Perhaps he may have considered all of them one by one and weighed judicially the stimulations and satisfactions which each had to offer. It is not impossible that he tasted many or all of these experiences. If he considered or tried them, he compared them all with one more attraction which offers its own peculiar delights to the minds of men. At thirty years of age, Fermat began to read mathematics. From that time on, there was no doubt in the mind of Fermat concerning the source of the deepest satisfactions in his life. It was mathematics; not mathematics which one learns for the purpose of using it in science or technology; not mathematics which one masters in order to teach it, or to publish it, or to debate about it, or to claim credit for its discovery; but mathematics as the revealer of law, the satisfier of the hunger to know truth and be free.

Fermat wrote little and published almost nothing during his life, but a few persons knew of his epoch-making contributions to mathematical knowledge. As time went on, more and more persons knew of his work, his freedom from mathematical error, and his "last theorem," the world famous proposition of intriguing simplicity which no one has yet succeeded in proving.

The experience of Fermat is one more example of the inherent power of mathematics to capture and retain the interests of men and to bestow generous rewards upon those who devote themselves to it.

Maria Agnesi

In the year 1802, a group of scholars in Milan met at the home of Agnesi, professor of mathematics in the University. They had been meeting thus for a number of years, taking their turns at reading research and philosophical papers which became bases for general discussion. A frequent contributor was Professor Agnesi's brilliant daughter Maria. When she was fourteen years old, she delighted and astonished them with her first paper. This had been followed by many others which showed her rapidly maturing philos-

ophy. She was at once accepted as an equal among the scholars.

As they gathered on that evening in 1802, they anticipated another paper by the precocious child, now a mature woman. One of the scholars surrendered his long cloak to a servant and greeted his host. "Ah, Professor Agnesi, your daughter will read again this evening, of course?"

"How shall I tell you?" said Professor Agnesi. "Maria will never read again. She will appear among us no more forever. She will never again come and go through that door."

"What!" the scholars exclaimed. "You do not mean she is dead?"

"No, no," said Professor Agnesi. "Not dead, but alive no more to the concerns of this earth. Maria has vowed to live in absolute seclusion from this time forth. For years she has begged to be allowed to become a nun. I have stood firmly against this and have hoped against hope that maturity would bring to her the realization that she is by nature fitted for the scholarly life. Yesterday she retired permanently to her rooms for a life of meditation and prayer."

Thus at twenty years of age, Maria Agnesi withdrew from the world without taking the usual accompanying step of entering the church as a nun. She had to find her own interests and activities, to fix her mind on such thoughts as would seem to her to be the most complete compensation for all she was giving up. To what can one turn at such a time? One could do much worse than to do as Maria Agnesi did. She turned to mathematics. For fourteen years it was her companion in seclusion, one of the two great compensations for her loss of human contacts and the material interests of the world. She read it daily, did research to learn what had been done previously, and made original discoveries. It became her work and her play, her labor and her amusement, her sacrifice of ease and her full reward. Perhaps no one ever lived so intimately and exclusively with mathematics for so long. Maria Agnesi made the great test of the subject as a power to fill a life with beauty and truth when every other interest (religion alone excepted) has been severed.

One of the special curves studied in analytical geometry bears the name of Maria Agnesi; it is called the Witch of Agnesi.

When she was thirty-four years old, Maria was persuaded to leave her seclusion to teach for a while her father's classes at the University. Then she devoted her life to the work of the church which she served for forty-seven years. Perhaps it is not an unreasonable surmise that during that half century she sometimes put aside beads and prayer-book and sought again the consolations of mathematics.

Sonja Kowalewski

One of the outstanding literary events of the year 1889 was the publication of a novel by a new writer, Sonja Kowalewski. Although the novelist was a Russian, the novel was not first printed in that language. It was issued simultaneously in Swedish and Danish. Soon it was translated into the novelist's native Russian and, as sales and appreciation of the book mounted, it was translated into other languages. Literary critics unanimously declared that Kowalewski was the equal of Russia's greatest writers both in thought and literary style. A brilliant literary career was predicted for her, but Sonja Kowalewski never wrote another novel.

Kowalewski was thirty-eight years old when she achieved sudden international fame with her one literary production. Why had she waited so long to exhibit this remarkable talent, and why did she at last turn to fiction? There is one answer to both questions, a one-word answer; it is mathematics.

As a young girl, Sonja Corvin-Kroukowsky was exceptionally beautiful and exceptionally talented. So far as we know, she was not seriously tempted to follow any career making use of her asset of beauty. However, having equal talent in literature

and in mathematics, she hesitated briefly between devotion to one career or the other. Before she was eighteen years old, the pull of mathematics won. It even compelled her to enter into an early marriage of convenience as a necessary prerequisite to study abroad. As Sonja Kowalewski, she went to Germany and studied privately under Weierstrass in Berlin. This noted mathematician and teacher cultivated her, encouraged her, and goaded her into full development and application of her powers. With some interludes of idleness she moved from one triumph to another until she became the outstanding woman mathematician of the nineteenth century and one of the greatest of all time.

At the age of thirty-eight she won the Bordin prize of the French Academy of Sciences in competition with the best mathematicians of the world. Not only that, but her paper was of such exceptional merit that the prize was doubled.

The strain of producing the paper and the excitement of winning overtaxed Kowalewski, and the flesh cried out for rest. As a means of relaxation she turned to fiction with the result already noticed. After producing one novel, she returned to her mathematics to which she remained devoted until her death two years later.

Here is another example of the power of mathematics to win the interest and devotion of an individual in even contest. It offers all the romance fiction can give, all the thrill, all the beauty.

Conclusion

There are those who believe that mathematics is for mathematicians. There are also those who believe that a mathematician should devote himself to mathematics exclusively because the nature of his major interest unfits him for any other activity. The evidence (a small part of which is cited in this paper) is against both of these conclusions. Mathematics is for the millions, including lawyers, statesmen, architects, recluses, and artists. On the other hand, mathematicians often step out of the role to become successful statesmen (Leibnitz), inventors (Huygens), writers (Carroll), humanitarians (Nightingale), and masters of the mint (Newton). These facts seem to indicate conclusively that some knowledge of mathematics is and should be only one form of general culture. It is high time that it should be accepted as such. Why should a cultured individual regard it as essential that he know that Lewis Carroll is the author of *Alice in Wonderland* and not trouble to know that he was also an accomplished mathematician? By what line of reasoning can one come to regard it as culturally necessary to know that Christopher Wren was a great architect and not know that he was also a recognized mathematician? Or why should one blush to have to confess that he knows nothing about poetry and smile complacently on the admission of complete ignorance of geometry?

If mathematics is to become an element of general culture, teachers of the subject will need to present it in such a way that it can make its cultural contribution. This can be done by conscious attention to the enrichment of teaching, one phase of which is the devotion of a little time to acquaint pupils with the experiences of persons who have studied mathematics as they read their novels or their Bible, for intellectual, cultural, and spiritual benefits.

Mathematics Teachers Train Thrifty Citizens*

By Jarvis M. Morse
Education Director, U. S. Savings Bonds Division, Treasury Department, Washington, D. C.

I am not a professional mathematician. You might consider me a practicing mathematician. Ever so many nights a week during the school year young Johnny, who is reviewing basic operations for Junior High, says, "Hey, Dad, are these things right?"

One of them is the simple problem—$54.60 is 3 per cent of what number? Johnny's answer of .0005 floors me. You would have known instantly what the trouble was, but it took me a few head-scratchings before I realized how he came out with that infinitesimal figure instead of $1820.

From this fraction of a mill at bed time, I plunge the next morning into the Treasury Department, where they deal in somewhat larger figures.

The national debt reached an all-time high of $279.8 billion in February 1946. Three years later it had been reduced by $27 billion. What per cent reduction was that? At the present time the debt is about $257 billion. What per cent is that of the post-war peak?

In December 1940, a year before Pearl Harbor, *individuals*, as distinct from corporations, held 21 per cent of the national debt. Now they hold 28 per cent. What does this increase mean in the management of the national debt? These are sample items in the practical mathematics surrounding my livelihood.

Before we go into the question of why there is a School Savings program, or how mathematics teachers train thrifty citizens, I should like to say a few words about the Savings Bonds Program in general.

The Savings Bonds Program is not a war baby. Low-denomination Savings Bonds—registered, non-transferable, redeemable at the Treasury at fixed rates, were first offered to the public March 1, 1935.

Their fiscal purpose was to help the Treasury at that time with deficit financing. There were three collateral objectives:

1) To instill the habit of thrift into the minds of the American people;
2) To educate the people with respect to securities issued by the government;
3) To align people more closely with the government. Investors in Savings Bonds became more aware of their personal stake and continuing interest in governmental policies, particularly financial ones.

The Series A Savings Bonds of 1935 were followed in 1936 by Series B, then Series C in 1937–38, and Series D from January 1939 to May 1941.

These bonds, popularly called "Baby Bonds," were sold through some 14,000 post offices and promoted by direct mail by the Treasury. Sales increased from about 277 million for the fiscal year 1936 to $1,100,000,000 in fiscal '40.

For some time before World War II overtook the United States at Pearl Harbor, Treasury officials studied means of diverting excess purchasing power from the consumer market, and thus reducing inflationary pressures appreciably in 1940–41. The decision to continue with low-denomination bonds suitable to the great mass of moderate investors was based on the principle of wide distribution of the debt:

> To the extent that loans are made out of current income, and are widely distributed among the people, the inequities arising from the final distribution of the burden are minimized, and widespread ownership of the national debt becomes a stabilizing factor of great importance.

May 1, 1941, the foregoing Savings

* Read at the Annual Meeting of the National Council of Teachers of Mathematics at Chicago, Illinois, April 14, 1950.

Bonds were replaced by three new series. The Series E Defense Savings Bond (renamed War Savings Bond in June 1942, and then restored to U. S. Savings Bond in 1946) was essentially the same security as the preceding bonds—beginning with a $25 denomination (issue price $18.75), registered, non-transferable, redeemable at the Treasury at fixed rates. For larger investors, Series F and G bonds were added to the offering. Series E and F are appreciation bonds; Series G pays interest semiannually. All are still actively on sale to the public.

In order to raise the large sums needed to meet military expenses 1942–45, the Treasury offered other securities (notes, certificates, and negotiable bonds) in eight War Loan drives, but Savings Bonds remained continuously on sale between drives as well as during them.

After December 1945, the bond program was reorganized on a peacetime basis, under the management of the U.S. Savings Bonds Division, charged with the responsibility of promoting the sale of E, F, and G Savings Bonds and Savings Stamps to the public. This is the Division which promotes the School Savings Program to aid schools give training in thrift and wise personal money management.

What about this national debt of ours? Except for a brief period in the days of Andrew Jackson, this country has always had a national debt.

Beginning with the Revolutionary War which accomplished our birth as a nation, this country has five times incurred a sudden expansion in its national debt to finance wars. It has been our settled policy to pay down our war debts as rapidly as possible. The situation which exists as a result of World War II is not different in kind from that following other wars, but it is markedly different in degree.

As already noted, the national debt reached a peak, an all-time high, of $279.8 billion, in February 1946. In the next three years, it was reduced some $27 billion. It has increased slightly in the past year.

The national debt is less volatile, and easier to manage, when it is held by a large number of individuals rather than concentrated in the hands of banks or a few large holders. Individual bond holding also lessens the danger of inflationary credit expansion. Hence, the Treasury has continued the active sale of Savings Bonds.

I have already mentioned the increase in *individual* holding of Government securities from 21 per cent in December 1940 to 28 per cent today. Since the Treasury has not offered long-term bonds since the war the increase now in individual holdings is due to the excess of new Savings bonds sales over maturities and redemptions. At the end of the war, the total holdings of all types of Savings Bonds was $46 billion. Today the figure stands at about $56 billion. The increased individual holding, both in dollars and per cent of total, is not only of significance to the management of the national debt but indicates a strengthening in individual habits of thrift.

The over-all Savings Bonds Program is not for the purpose of increasing the national debt, but to help the Treasury manage the debt—to keep it spread out in the hands of as many individual investors as possible.

Now what about School Savings? Your students, and no doubt their sons and daughters, are going to have to live with this or some national debt. It is most desirable that they know something about it. Further than that, ownership of a piece of the debt gives the owners, young or old, a real connection with their government. And still further, from a personal point of view, a Savings Bond is a most desirable investment for anyone. Series E Bonds are safe—registered at the Treasury, replaceable in case of loss or theft, bearing a good interest (2.9 per cent compounded semiannually when held to maturity, ten years), redeemable if necessary after 60 days from date of purchase.

Now let us get down to cases. School Savings can acquaint your students with Government securities and the national

debt. School Savings offers them the opportunity to make a desirable investment for themselves—for further education in college or business school, for setting up in business on their own, for marriage and a home.

These are ends eminently worth promoting. What does the Treasury do to help you? Some, I know, are afraid that School Savings is a complex program, consisting of many parts from A to Z, that must be followed in toto, perhaps to the detriment of other areas of learning.

This is not so. The Treasury offers you a few aids, any one or several of which you may use to suit your own particular circumstances. Generally we have:

The *School Savings Journal* (semi-annually)
Lessons in Arithmetic Through School Savings (elementary grades)
Teaching Mathematics Through School Savings
Budgeting Through School Savings
School Savings in the Social Studies
Plus one or two plays and posters.

It is in the field of mathematics that the closest correlation can be made between the theory of saving and the actual practice of thrift, to give meaning to the theory.

Two of these booklets we offer, the *Arithmetic* and the *Mathematics*, were prepared by a member of your organization, Mrs. Irene Reid of Miner Teachers College, Washington, D.C.

I do not claim that they are perfect. Perhaps they should be further broken down by age or comprehension levels, to make them just a bit easier for busy teachers to fit into the regular curriculum.

But I do think the suggested problems are practical. When I was in school back near the beginning of the century, we had such neat arithmetic problems as:

If a farmer can scythe $\frac{1}{3}$ acre of hay in an hour, and his son half as much, how long will it take them working together to finish a 5-acre field?

If the average hen lays 200 eggs a year, and the average selling price over the year was 40 cents a dozen, how much will a poultryman realize in a year from a flock of 230 hens?

These were very practical and meaningful then. I lived in sight of hay fields, and my sister and I earned pin-money from some 50 hens, plus the roosters fattened for Thanksgiving and Christmas.

Conditions are different now for many of us. A great many of our students hardly ever see a horse eating hay no matter what the rate of cutting, and eggs come in pasteboard boxes. You like and need practical and meaningful problems. Those dealing with the amount of paper necessary to paper a room of certain dimensions may aid those few who become paperhangers. Some others may come in handy for prospective grocers or druggists. But whether today's students grow up to be interior decorators or grocers or retailers or what not, they, all of them, are going to have to manage their own money. So I think our School Savings problems are eminently practical. Take this one from *Arithmetic*, page 18:

John wants to fill his 25-cent Stamp album so that he can buy a Bond. If he has 52 Stamps and his father gives him money enough to buy 8 Stamps, how many more does he need to fill his book which holds 75 stamps?

Or from *Mathematics*, page 13:

A worker earns \$64.50 every 2 weeks. He allots \$6.25 through the Payroll Savings Plan toward the purchase of a \$25 E Bond. In how many pay days will there be enough allotted to purchase one Bond at issue price of \$18.75? How many Bonds would this worker receive in one year?

Or this one from *Budgeting*, page 9:

List some of the things you would like to save for. Which are the most important? Which should you have for your improvement or your future? Which would you like very much to have? Which can you really do without?

I believe you people hold that the best problems arise from real situations. The weekly Stamp and Bond day activities offer a wealth of problem material in a real situation—the checking of sales and receipts, checking of balance in the revolving fund for Stamp purchases, computation of percentages of student, or classroom, participation, illustration of the latter by graphs, and so on.

So I earnestly recommend School Savings to you as a very desirable project. It contributes to foresightedness, responsibility, and good citizenship. It takes little time. Under the minimum of faculty supervision, students can manage the savings program themselves, the regular investment in Savings Stamps and Bonds, perhaps through a set period on a designated day each week. The teaching aids, to encourage the practice, fit into regular curricular subjects, mathematics especially.

Copies of the three School Savings aids I have just mentioned may be procured, free for the asking, from the Savings Bonds office for your own state. Or you may write for the same by sending a postal card to the U. S. Savings Bonds Division, Treasury Department, Washington 25, D.C.

Thrift in individuals will encourage thrift in government. There is danger that too many of our citizens today fall into the easy habit of looking to government to solve all their problems—of illness, unemployment, and old age. School Savings can help train a thrifty generation in the good American tradition of self-reliance, of ability to stand on one's own feet in security and independence. I solicit your help, for the good of all of us.

A "Self-Starter" Approach to Fractions

By Jack V. Hall

Central Washington College of Education, Ellensburg, Washington

The development of the Model T Ford was a highlight in the history of transportation. Yet, as most of us fondly recall, it had shortcomings. The early models had to be cranked to start. Model T Fords are scarce in the rapid traffic of today. In contrast, "Model T" arithmetic teaching procedures are still in evidence in many of our classrooms. Too many teachers of arithmetic are still "cranking children's mental motors" by forcing them to memorize mathematical principles and generalizations stripped of meaningful introductory experiences.

The invention of the self-starter revolutionized the "Model T." No longer was it necessary to get out and crank our cars on cold mornings. The "self-starter" of a modern arithmetic program is the discovery approach. Rather than laboriously "cranking children's mental motors," resourceful teachers are making use of (1) direct experiences, and (2) manipulative materials as children's "self-starters" in the discovery of important mathematical understandings. The purpose of this article is to illustrate items (1) and (2).

I. Illustration of a Direct Experience

Developing Fraction Concepts with Pupil-Made Football Flags

Any school environment is teeming with potential direct quantitative experiences ready to be tapped by alert teachers. Specifically, one area rich in number possibilities is the playground. The sixth grade boys in the College Elementary School at Central Washington College of Education, like most boys engrossed in the excitement of a touch football game, sometimes have difficulty deciding whether or not the ball carrier has been touched. It was agreed that flag football might be a solution, for by inserting a canvas flag under each boy's belt, any member of the defensive team could grasp it as tangible proof of contact.

Kenneth volunteered to bring an old canvas tarpaulin to school as material for making football flags. A real arithmetic problem arose when the children gathered around the tarpaulin spread out on the floor. Rather than "cranking their mental

motors" by phrasing the problem for the class, the writer let the children use their "self-starters" by asking, "How can we measure and cut our tarpaulin to make enough flags?" Since the class had no experience in computing area, several impractical solutions were offered. Nancy recalled the paper folding demonstration, as a review of fractional concepts, in which the class participated earlier in the autumn. She suggested, "Let's fold and cut our canvas just like we did with the paper to explain fractions." John objected because, "The folds would make halves, fourths, eighths, sixteenths and thirty-seconds. Thirty-two pieces of canvas would be too many. There are only twenty-two players on two teams." Sandra remarked, "Yes, but remember that the canvas has three tears in it. Some of the pieces won't be any good." Paul added, "We might need some spares in case any flags are lost." The group agreed that Nancy's solution was best. A committee of six boys and girls were chosen by the children to make the flags. Kenneth was appointed chairman since he provided the canvas. First, they folded the canvas, pressing each fold which left a crease to be marked with chalk. Second, the canvas was cut with large shears along the chalk lines. Third, each flag was folded in pleats an inch in width. Fourth, each flag was bound in the middle with friction tape, leaving the ends free to flare out as a good target for defensive players to grasp. Fifth, the friction tape was coated with shellac for permanence. Following these activities with flag making, the children stated and solved fraction problems that related to these direct experiences.

We lost 4 flags because of tears in the canvas. What fractional part of the entire canvas was lost? ———

We need 22 flags for the two football teams. This is what fractional part of the entire canvas? ———

John's flag is what fractional part of the entire canvas? ———

How many spare flags will we have in case some flags are lost? ——— What fractional part of the entire canvas is this? ———

II. Illustration of Manipulative Material

Pupil-Made Individual Fraction Boards

Busy teachers seldom have time to construct their own manipulative aids to increase children's understandings of quantitative concepts. Furthermore, concrete arithmetic materials now available from commercial sources leave much to be desired because (1) they are often expensive, (2) they do not always meet the needs of the children, and, (3) they are usually designed for group demonstration in which the child performing the manipulation receives the most benefit. These reasons prompted the writer to launch a project in which the sixth grade class made their own individual fraction boards.

First, the children determined the amount of flannel cloth needed for the project and selected a committee to make the purchase. When informed that colored flannel was expensive, the class agreed to dye inexpensive flannel cloth. The following directions were adapted from the instructions contained in a box of dye:

1. Dissolve the dye in a small pan of boiling water.
2. Clean sink thoroughly.
3. Wash flannel cloth in warm water and rinse.
4. Drain the sink and rinse.
5. Fill the sink with enough water to cover the flannel.
6. Put two tablespoons of salt in the water to set the color.
7. Pour the dye in the water and stir thoroughly.
8. Dip the flannel in the dyebath and keep it moving constantly for about 15 minutes to make sure the flannel does not streak.
9. Remove the flannel and rinse well in clear, cold water, until the rinse water runs clear.
10. Gently wring out the excess water.
11. Place flannel on wrapping paper over the radiators to dry.

Second, the flannel boards were constructed according to these directions:

1. Bring scrap plywood $12'' \times 18''$ or larger from home, if possible.
2. Cut plywood boards $12'' \times 18''$.
3. Sand all edges so that splinters will not come through the flannel cloth.

4. Cut yard wide cloth in 21" lengths.
5. Cut each 36"×21" piece in half to make two 18"×21" pieces.
6. Stretch flannel tightly over the width of the board and tuck edges under to prevent raveling. Staple the middle of each end first. Then staple the corners of each end.
7. Now stretch the flannel tightly over the width of the board and do each side just as you did the ends.
8. Staple and glue the middle of a 10"×15" manila envelope on the reverse side of the board to store fractional parts. Do not staple or glue the edges because it will be difficult to insert your hand without tearing the envelope.

Lastly, the children assembled the fractional parts, by doing the following steps in sequence.

1. Collect lightweight cardboard boxes from stores.
2. Measure and cut the 6 inch cardboard disks.
3. Using cardboard disks as patterns, outline circles with chalk to make the largest number of circles because flannel is expensive.
4. Cut a 7 inch flannel square containing the 6 inch circle.
5. Secure the flannel square on your drawing board with thumbtacks.
6. Spread clear rubber cement thinly over one side of each cardboard disk with your brush. Remember to cover the edges with cement. Work rapidly as rubber cement dries quickly.
7. Work in teams of two: one person cementing the cardboard disk, while the other person cements the flannel.
8. Allow the cement to dry for 5 minutes.
9. Lay the cardboard disk on the flannel. Be sure to place them within the area of the chalk circle. Press the cardboard disk with your fingers from the center outward to the circumference of the flannel circle.
10. Remove the thumbtacks and put the disks under heavy books.
11. Trim the flannel as close to the cardboard as possible.
12. Repeat steps in cementing contrasting colors on the reverse side of each disk.
13. With the aid of patterns, cut the disks into halves, thirds, fourths, sixths, and eighths.

Use of Individual Fraction Boards

The "pie" figure, as represented by the fractional parts, is possibly the best approach to the understanding of (1) the relationship of a part to the whole; (2) the relationship among the parts to the whole; and, (3) the processes of addition, subtraction, multiplication and division of fractions. Manipulating the fractional parts enables the child to use his "self-starter" to discover these understandings. By reversing one or more fractional parts to a contrasting color after a unit has been constructed, attention is drawn to individual parts for identification or comparison with the whole.

Detailed examination of several fractions in concrete form may be carried on by each child individually under the direction and guidance of the teacher. To illustrate, 4/6 can be separated into 1/6 and 3/6; 2/6 and 2/6; and 3/6 and 1/6 (subtraction). These parts can then be combined (addition). In the same manner 1/2 of 4/6 can be discovered (multiplication). "How many 2/6's in 4/6?" (division) can also be represented. Reduction and equivalents, $2/6=1/3$; $3/6=1/2$; and, $4/6=2/3$ are proved to the child's satisfaction. Adding 4/6 to 4/6 establishes his concept of improper fractions and mixed numbers.

We must always be mindful of utilization, for, if used improperly, this device could conceivable be reduced to the level of a plaything or gadget.

Prerequisite to Meaning*

By GERTRUDE HENDRIX
Eastern Illinois State College, Charleston, Illinois

ONCE upon a time not too long ago, when factoring was a formal and formidable part of elementary mathematics courses, we all learned a definition for prime number. Perhaps you as well as I can remember someone who could recite or write, "A prime number is a whole number greater than one which has no factors other than itself and one," but who could not find answers to such questions as: "What is the smallest prime number greater than 100?" or "What is the largest prime number less than 125?" Some of our teachers thought they were preventing superficial memorizing of the definition when, instead of asking us to quote it, they asked for five examples of prime number, but again the question fails as a valid test for the concept. To be sure, the question exposes one who can remember only four of the examples he once memorized, but one who can remember as many as five is as well off on that test question as he would be if he really had the concept. The problem is, how can we direct learning so that such superficiality, waste of time, and intellectual dishonesty are prevented rather than promoted.

Our books may not mention prime number today, but occasionally the need for the concept bobs up anyway. Not long ago a high school boy was trying to prove that the square root of seven is irrational, in the same way that Euclid proved the square root of two irrational. He came to the part of the proof in which he asserted that since p^2 was divisible by 7, then p also was divisible by 7. He said, "Now I don't know just how to state the reason for this, because there are some numbers for which it works, and some for which it won't work. Seven is one of the kind for which it *will* work." The teacher asked him to name some others of the kind, and the boy named several primes. To the class the teacher said, "The smallest number of this kind is 2; the next one is 3; the next 5; then 7; the next one after 7 is 11. Any time that you can name the next one before Richard and I do, call it out. We'll start again and go slowly: two three, five, seven, eleven, thirteen, seventeen, nineteen—." At this point one pupil broke in with, "Is the next one twenty-three?"

"Yes. What is the next one after that?"

"Twenty-sev—No! Twenty-nine!"

He had it. Very soon almost every member of the class seemed to be finding the next prime independently. Something subverbal—something organic, if you please—had happened to them. They had come into possession of awareness of a class, or an unawareness of the relation which draws prime numbers into a class. They had the *prerequisite to meaning* of the term. They were ready for the name. They were told that such numbers are called prime numbers.

I would like to call that subverbal, organic, *dynamic* state of awareness the possession of the concept, but most psychologists, linguists, and philosophers today do not consider the concept complete in the person's mind until he has attached a symbol to it. This failure to recognize that awareness of an entity is independent of the existence of a symbol for the entity, promotes pedagogy that is not only wasteful, but often harmful.

May I go back for a moment to that eleventh grade class. I said *almost* every member "had it." One pupil still seemed to be groping. The teacher asked her to start over again and name the prime num-

* A paper read at the St. Louis meeting of the National Council of Teachers of Mathematics on July 3, 1950.

bers beginning with two. She said, "Two, three, five, seven, eleven, thirteen, seventeen—", and stopped. "I've forgotten what came next," she said. She was told to write the seven numbers she had named in one list; to write the other numbers between 1 and 17 in another list; and to examine the two lists for differences. Presently, she interrupted the class discussion to say, "I see! the next one after 17 is 19, and the next after that is 23; then—um—29. I see how they were getting them now." May I repeat, only then did she possess the prerequisite to meaning for the term, "prime number." A moment earlier, "prime number" was for her a *proper* noun, identifying an arbitrarily selected finite group of discrete objects. She was not aware of the property which draws prime numbers into a class. It seems to me quite proper to say that *her* selection of 19, 23, and 29 as the next three members of the class was behavioral evidence that *she* had the *meaning* of "prime number," because at that time, they all had been told the name. But what about all the others who could do the same thing before they *knew* any name for prime numbers? Theirs was the same behavior, but it could not have been behavioral evidence of *meaning*, because the entity called meaning is a triadic relation—a relation involving a person, a referent, and a symbol. It seems indisputable to me that for all but one member of that group ability to select prime numbers was behavioral evidence of awareness of the existence of such a class of numbers, not behavioral evidence of meaning. It became behavioral evidence of *meaning* only after a name had been attached to the class.

I could spend hours describing other examples of this kind of thing that I have seen and heard in the classroom. These phenomena play havoc with the current practice that *the* way to develop a concept is to show example after example of the thing, at the same time repeating a *word* for the thing. I realize that concepts *can* be (and, sad to say, usually *are*) developed in that way. The development is successful if the teacher is not deluded by glib users of the new word citing memorized examples, but instead, meets Henry Van Engen's charge to demand behavioral evidence of meaning. Even so, it is a costly fact that this process, by which we acquire most of the words of our mother tongue, fosters the formation of a false generalization concerning the nature of language: repeating the word with each example during the concept-forming experience produces a false subconscious if-then relation between the word and the thing. We have been a long time realizing that subverbal awareness of a class, or a property, or a relation *had* to be in *some*one's mind before anyone could have thought of inventing a word for it, anyway. In the natural order of events, the abstraction forms first, and *then* a name for it is invented.

Before going on, may I report that in much less time than it has taken to tell about it, the members of that geometry class, in which a boy was trying to prove the square root of seven irrational, stated their missing lemma, "If a prime number is a factor of n^2, then it is also a factor of n," and the proof was completed. Some of you will be relieved, perhaps, to hear that on the following day during a review the pupils also formulated a good definition of *prime number*. At this point some may want to ask if all that time was not wasted for the boy giving the proof, since he already had the concept at the beginning of the hour, and needed only the name. I think not. He participated in the discussion with attention and frequent contribution, helping out with the teaching whenever the teacher could draw him into it. He was learning how to make an idea of his own clear to others. I strongly suspect that many a great idea dawns and dies in this world simply because the gifted person who has the creative insight in the first place cannot get his vision across to enough of the rest of us for his idea to be tried.

We come now to the kind of generalization represented by if-then statements—the universally quantified conditional, logicians call it. Here it seems to me we *must* differentiate between meaning and understanding.

To distinguish between meaning and understanding, let us see (1) how generalizations of the if-then type are used, and (2) how the word "understanding" is used. The simplest kind of universally quantified conditional takes the logical form, "For every x, if x has the property (or combination of properties) P, then x has the property (or combination of properties) Q." Let us examine the example, "If a large enough object is between us and the sun, then the sun is hidden from sight." In other words, "for every x, if x is a situation in which a large object is between us and the sun, then x is a situation in which the sun is hidden from sight."

Awareness of such a generalization yields at least three kinds of transfer power. To the astronomer observing motion and relative position of sun, moon, and earth, the generalization, coupled with the particular, "at such-and-such a time the moon will be between the sun and a certain portion of the earth's surface," yields power of prediction: "At that time and place the sun will be blotted from sight." This power of prediction is one kind of transfer power to be derived from a universal if-then.

A second kind of power comes when an organism can produce an event of the if-class, thus obtaining an event of the then-class. The astro-physicists who recently produced an artificial eclipse of the sun, enabling themselves to take their famous motion picture of the sun's atmosphere with its spectacular storms and explosions, used the generalization in this second way.

A third kind of transfer power available to one who knows the generalization is the poise-yielding power to explain to oneself an event of the then-class. In this power lies the difference between human behavior in an American village during an eclipse of the sun, and human behavior under the same circumstances in a village in India. We say *we understand* the event. It seems to me that to *understand* an event is to see how it might have been predicted. In the same way, dynamic possession of the generalizations of geometry enables one to note with *understanding* a myriad of daily events which otherwise either would not be noticed or would appear as troublesome mysteries.[I believe apologists for mathematics in general education make much too little of this poise-yielding power of elementary and secondary school mathematics to all who possess their generalizations on a dynamic level. The person who knows the converse of the Pythagorean theorem *understands* what a carpenter or stone mason is doing when he marks off three- and four-unit segments on the sides of an angle and compares the distance between their outer ends with a five-unit segment. The understanding observer is in some subtle way more alive at that moment than one to whom the carpenter's act is a mystery.]

Seeing how an event might have been predicted may or may not involve ability to interpret a sentence—in other words, understanding of an event may or may not involve *meaning* of a sentence. But it *does* involve the *prerequisite to meaning* of a sentence that *might* be stated. The generalization which provides the satisfying feeling of explanation or understanding of an event, may be held on a purely subverbal level—a universal if-then of which the person became aware at sometime in his past, but which he has never formulated into any kind of language. John R. Clark's reports of the arithmetical power of some army illiterates during the last war are rich with behavioral evidence of mathematical generalizations held on this dynamic subverbal level. So though understanding of an event *may* involve interpretation of a sentence which states a generalization, it *need* not do so. But it does involve the prerequisite to meaning of such a sentence.

Let me digress again to emphasize that this prerequisite to meaning *can* be built up *after* one has received the sentence ready made from mouth or printed page. That is what you and I *had* to learn to do; and any educated person must learn to do it to some extent. That is what the upper five per cent of the high school students who did show evidence of transfer in Thorndike's studies had done. One sufficiently skillful in interpreting sentence structure as well as referential symbols, can read a sentence which expresses a generalization, and then construct or find enough examples of his own to make the generalization an organic part of himself—that is, to acquire the subverbal thing prerequisite to meaning of the sentence. Most of the boys and girls in our schools have no such mastery of the tool called language. For them, teaching needs to be planned so that the thing comes first as an insight. Formulation of adequately representative sentences can take place later. Not that this formulation is to be belittled. It is essential for testing a generalization by proof, for recording the generalization, and for communicating with others who possess the same generalization. [I am aware of a fourth purpose, also, for which formulation of newly discovered generalizations must be carried out, but its discussion does not belong in this paper.] But we must stop confusing the process of becoming aware of an if-then relation, with making a sentence that expresses that relation. It is the subverbal awareness which yields the power of transfer, whether that power be prediction, problem-solving, or explanation. Those of us who saw and heard Howard Fehr's demonstration lesson at the Chicago meeting of the National Council last April saw a masterful and exhilarating example of a teacher promoting vivid generalization as awareness. Each time, evidence that the magic thing had happened was ability to give the next term in the sequence—that is, deductive application of an unstated generalization just formed.

Now finally, a word to those who maintain that to understand a *sentence* is the same thing as to interpret the sentence—that is, to know its intended meaning. Again, let us see how the word *understanding* is *used*. A pupil reads, "An angle inscribed in a semicircle is a right angle." The teacher asks her to illustrate—or to verify—the sentence by a drawing. The pupil goes to the board and presents behavioral evidence that she knows what the sentence means. But she turns and wails, "I know it *looks* like a right angle, but I don't see why it *has* to be. *I don't understand it.*" Her drawing reveals unquestionably that she has interpreted the sentence; she knows its *meaning;* but she doesn't *understand* the generalization for which the sentence stands. You and I both know whence comes the mental poise, the satisfaction, the *understanding,* if you please!—which she seeks. Experiencing the deductive proof is the only way. To understand a generalization is to see how it might have been derived—that is, predicted—from other things already known. One of the most difficult situations of this kind in our present curriculum arises with the generalization, "If a product is obtained from factors one or more of which are approximate numbers, then the number of correct figures in the product cannot be expected to be greater than the number of significant figures in the least accurate factor." What teacher has not had some pupil, courageous in his as yet unabandoned quest for intellectual integrity, say, "Yes, I know it says that the product of 14.1 and 37. should be rounded to 520, but I don't *understand* it." The correctly rounded answer is behavioral evidence that he knows what the sentence *means;* but he doesn't *understand* it. It is possible to set up an experiment through which the generalization above can be discovered—that is, form on a subverbal level. But this prerequisite to meaning of the sentence is not the same thing as understanding of the generalization represented by the sentence. Understanding requires the

concept of relative error, the proof that relative error of a factor is carried over into a product, and discovery of the relation between relative error and number of significant figures.

I now invite you to come with me on an adventure in conjecture. We all know of Pavlov's famous experiments interpreted as evidence of a thing called conditioned reflex. Most people think that association—mere co-existence of the sound of the buzzer and meat in the dog's dish—caused the dog to react to the buzzer as if it were the meat—that is, for saliva to begin to flow at the sound of the buzzer. The cause was thought to be conjunction, an "and" relation between the two classes of events. Forget that, and suppose for a moment that it was an if-then relation of which the dog had become aware. "For every t, if t is when the buzzer sounds, then t is when there is meat in my dish." At the next sound of the buzzer, the generalization yielded a prediction: "There is meat in my dish!" In his imagination the dog can see the meat. And so—well, suppose this were 11:30 A.M. instead of just after lunch for you and me. And suppose that from the next room we could hear a sizzle that sounded like a prime sirloin steak being broiled over charcoal. Well?

Let us go on. Suppose that an outstanding characteristic of organisms of the animal kingdom is sensitiveness to if-then relations between classes of events. For example, a three-year-old filly whom I held by leadstrap out on the prairie during a thunderstorm two years ago was at first terrified by each great clap of thunder. Not that she had never heard storms before, but this was different, roaring up in strange surroundings while a garage wrecker and mechanic hoisted the trailer to repair a blown-out tire. As the storm drew nearer, the filly began to notice the great streaks of lightning, too. She would throw up her head in interest at each jagged flash, only to jump almost out of her skin at the ensuing clap of thunder. But not for long. Suddenly something happened to her: After each flash of lightning she began to lower her head and draw herself into a crouch, braced to endure the clap of thunder—behavioral evidence of prediction made possible by awareness of an if-then relation between flashes of lightning and claps of thunder.

These subverbal generalizations in man or beast are tremendously dynamic. When a situation to which they apply comes along, they "turn on" in transfer behavior in spite of ourselves.

Now let us consider cases in which events of the then-class are events produced by the organism. The thing then shows itself as what we call a habit.

Next, think of the cases in which events of the if-class are events produced by the organism. If events of the then-class are undesirable to the organism, transfer behavior shows itself in avoidance of events of the if-class. If events of the then-class *are* desirable, transfer behavior is revealed by the problem-solving phenomenon: The organism produces an event of the if-class in order to obtain an event of the then-class. We are now very close to the invention of symbols.

Some organisms have become aware of the class of if-then relations, an abstraction on the next higher level. Application of this awareness enables him to set up an artificial if-then relation between a class of events or objects external to himself, and another class of events or objects which he himself can produce. "If I see a man with a gun, I go 'Caw-caw'." Add to this the converse relation, "If I go 'Caw-caw', I see a man with a gun," and we have a symbol. I believe, that a symbol is simply this: A class of objects or events producible by an organism, and bearing an artificial if-and-only-if relation to some other class (or property, or relation), the said if-and-only-if relation being set up and maintained by the organism itself.

I have gone much farther than this, but this is enough for one's first dose, perhaps. I shall close with a few hints of adventures

ahead. From conjectures above, emerge hypotheses which explain the difference between language signs and symbols in general; a definition distinguishing words from other language signs; a theory of learning which I believe explains everything for which other theories have been thought necessary; a hypothesis concerning origin and nature of language which I believe supplies the missing link in anthropology; a new explanation of the loaded-word phenomenon in basic communications; a new insight into foundations of mathematics which resolves the seeming contradictions between the logicist, intuitionist, and formalist schools of thought; and finally, the emergence of a new theory of instruction which explains what has always gone on in the best of teaching, and explains it clearly enough for the art to be passed on more effectively than ever before. To me, this final fruit is the richest of the lot. It is not an anti-climax to name this last point last.

Membership Message

This is to announce to you definite plans in regard to our Membership Honor Schools for the year 1950–51. Your National Council has been very much gratified by your hearty endorsement of this special Membership recognition, and your expressed desire for its continuation.

We plan to publish two listings again this year—the first in the February 1951 issue of THE MATHEMATICS TEACHER—and a second listing in the May 1951 issue.

It is highly probable that many schools now have 100% membership in the National Council—or that all of their mathematics teachers but one are members. If you do not now qualify for this Honor listing, we are sure you will soon do so. May we hear from you to this effect very soon so that we may include your school in our first Membership recognition. Simply copy and complete the accompanying form and mail it to

Miss Mary C. Rogers
462 North Avenue, East
Westfield, New Jersey.

Such reports should be in our hands not later than December 1, 1950, to be included in the February listing. We have already heard from several schools. We shall be watching the mails for letters from you. Please accept our sincere thanks for your cooperation and helpfulness.

Membership Report to the National Council of Teachers of Mathematics

School ....

School Address ....

....

Number of Teachers of Mathematics in Your School ....

Number of National Council Members ....

Report Submitted by ....

....

Your Address ....

....

Mathematical Notes*

By JACK C. ROSSETTER,
Oak Park High School, Oak Park, Illinois

PART 1

we're glad to have you teachers here:
ah, such a handsome lot.
for honestly we much prefer
ones pretty to ones NOT.

we could talk round in circles;
on tangents we could fly.
but you can surely see—can't you
how we would love to try

to teach to YOU who once taught US
a thing or two or three.
we quote "describe the forest
where grows ge—om—e—tree."

but let us grow more sober now
and brush you up on FACTS.
true history we have for you.
(of rumor, true, it smacks!)

take Thales—quite a business man!
when olives on the trees
got ripe. no one could find a press!
old Thales put the squeeze

on olives, and his brother greeks!
free enterprise, our guess is,
tho "under-handed" some called him;
for HE owned all the PRESSes.

Pythagoras a coffee pot
tall, hot, and brewing juice
of coffee bean, did see; cried he
"at last: hy-pot-in-use!"

old Zeno had them going 'round
and gasping, all, for air;
"the tortoise, he proved trickily,
just CAN'T be caught by hare.

for tho the hare is twice as fast,
the tortoise too advances
and so stays always out in front!
why all those puzzled glances?

the sophists later came along;
of course you know what TheY are;
they're freshmen who have moved ahead,
by one year smarter, gayer!

* Presented by the Speaking Choir of the New Trier Township High School, Winnetka, Illinois, at the Banquet at the Twenty-Eighth Annual Meeting of the National Council of Teachers of Mathematics at Chicago, Illinois, April 14, 1950.

Hippocrates then came in view;
we say this with a wink!
hippocrates are folks who say
things different than they THINK.

then Plato, b.c. 4-2-9.
HE says just what he means!
the only time we hear of him
today, is, plato beans!

then Euclid! bugles! roll of drums!
quite famous, don't you see!
assembled LEAVES from others' books—
and made ge—om—a—tree.

(WE borrow some one's paper
and copy just a BIT.
we get not fame but flunks and blame!
oh history, how she's writ!)

then Hero—him we always mix
with someone else—say NERO.
one fiddled round while Rome burned down;
on these two we get zero.

old Bacon—he fits in some place:
logician, thinking man!
got more math put in schools; we think
this bacon was no ham!

da Vinci did most everything;
did pictures of the saints;
invented; messy homework, tho,
we bet, all smeared with paints!

Copernicus and Gal-i-lee-
o peeked up at the moon;
decided we went round the sun;
yep, got in TROUBLE soon!

Napier, no lumberman was he,
nor butcher, brother jones!
altho it's true he played with LOGS,
invented napier's BONES'.

Descartes of France should get a word;
from esquire praise deserves.
for descartes did his famous work
on—pardon us—some CURVES.

Pascal a famous man could be
but left math in the lurch;
one day his horses ran away
and pascal joined the church!

of Newton everyone has heard;
of course he now is dead.
an apple dropped down from a tree;
old newt sure used his HEAD.

of Leibnitz we are not quite sure
altho we've got a feelin
the calculus he used, he got
from newton, just by stealin!

of Riemann, Lobachevsky, too,
and their geometry,
we can but say, "too deep for us;
hope it agrees with THEE."

to Einstein, hair and violin,
we give our final nod;
tho understood by just two folks—
himself—and sometimes—God!

PART 2

Local Celebrities

but why should we just speak of those
who are not here today?
when your own group boasts great ones too
who stellar roles can play?

o.k., 'tis done; just lend an ear
and let the heavens hark
to gossip, news, of local boys—
like Austin, Schorling, Clark.

oak park, a village of some size
with nose in air, like boston,
gave not U.S. but this group its
first president, charles austin.

exuding math from every pore
and bursting out with knowledge,
is schorling, who at u. hi shines
and clark, at teachers' college.

there's breslich, not a man of vice
except as president.
and reeve, when editor we sought,
was sure from heaven sent.

and betz, past prez, and author too
of articles and books,
speaks english oh so fluently;
at clocks he seldom looks.

our secretary schreiber writes
our minutes by the pages
but will not answer letters tho
some folks break out in rages.

miss gugle, long in junior hi,
set ohi-o a-flame
while don the dull was helped by one—
miss potter is the name.

doc hildebrandt we all know well
for he was always pest'rin
the teachers in this area
to dash out to northwestern.

before you ladies rise and shout
and claim that we have missed her,
we certainly shall tip our hats
to doc's quite able *sister*.

all states contribute to our fame!
a sample from the west
is harry charlesworth, denver, who
on n.e.a. talks best.

miss woolsey, friend of junior hi,
and wren of tennessee
are chatting with christofferson,
just back from germany.

the east sends smith and mallory;
and oklahoma, hassler;
while schult and rankin run, at duke,
a mathematics dazzler.

with shuster, fame—approximates,
miss sanford, second ed,
and carnahan, a hoosier brain,
our poem slows its tread.

a toast to all—and countless more
we missed and hadn't oughtter;
ohio state will fill our cups
from fawcett—cool, cool, water!

Any similarity to persons living or dead is purely intentional.

j.c.r.

The President's Page

The National Council of Teachers of Mathematics is now a department of the National Education Association with offices at the NEA headquarters, 1201 Sixteenth Street, N. W., Washington 6, D. C.

At the time of the Baltimore convention in March of 1949, after several years of consideration, your Board of Directors voted unanimously to petition the NEA for departmental status. This action was approved at the business session of the convention, and the petition with the required number of signatures of members was prepared. The matter was then referred to the membership at large through The Mathematics Teacher, inviting reactions. (Refer to page 290 of the October 1949 issue.) At its next annual meeting in Chicago, the Board renewed its decision to ask for departmental status. The Delegate Assembly of the NEA, meeting at St. Louis last July, passed favorably on our petition.

In the event of this favorable action by the NEA, your Board of Directors had previously laid plans for setting up our headquarters in Washington. It was decided to separate the editorial functions and the business management of the journal. The Board appointed your president acting business manager for the year August 1, 1950 to August 1, 1951.

Making the change from New York City to Washington has involved careful planning and a great deal of work. I wish to take this opportunity to thank Mrs. Katherine Reeve Girard for the fine way in which she carried on the work of editor and business manager of the journal during the illness of her father and until the change could be made. In behalf of the Council, I wish to thank Dr. W. D. Reeve for his many years of most capable and devoted service as editor of The Mathematics Teacher.

Office space, office equipment, and many valuable services have been provided by the NEA. We are impressed with the friendliness and helpfulness of everyone in the NEA offices. I hope you will visit your National Council headquarters when you come to Washington.

In our brief time here we have found that the possibilities of greater services of the National Council as a department of the NEA are many. I am more confident than ever that this new relationship with other professional organizations will mean a greater National Council of Teachers of Mathematics, a more powerful force for the improvement of mathematics education.

We are devoted to the purpose of improving the teaching of mathematics. We are therefore most anxious to help the teacher. We have several new endeavors under consideration, but we would like to hear from you. We would like for you to tell us what you need in the way of services and help that your National Council can give. You will be given opportunity to tell us some of these things by means of short questionnaires which will appear in The Mathematics Teacher from time to time. Please answer them.

We know that good mathematics programs are necessary and that curriculum revisions are needed. We realize that certain surveys and studies would be helpful. The right kind of leaflets and pamphlets, frequently made available, would be of great help. The journal can be made even more useful, the Yearbooks should be made to fit a current need, permanent committees to work on various teaching problems could be set up, means should be provided for closer cooperation of the National Council with other national organizations, affiliated groups should receive various types of help. There seems to be no end to the many things that might be done, but what are the things that you, the teacher, need most?

It is easy to think up a vast number of projects and activities in which the Na-

tional Council could engage, most of which would require large sums of money. But, until our membership increases, as it *is* doing very rapidly, and until we can get the help of foundational grants, we shall have to restrict our endeavors to those in which our own members will devote much time and work, everyone of us helping whenever and wherever we can.

So, please remember that we do want your suggestions and recommendations, individual as well as group. Feel free to write us any time.

H. W. Charlesworth, President

Concerning National Council Yearbooks

A Request for Your Help

At the 1950 annual meeting of the National Council it was decided to appoint a committee to be known as the Yearbook Planning Committee. This committee was given the responsibility of acquainting itself with matters pertaining to the needs and wishes of members relating to the types of yearbooks which they would like to have published and to take steps to have these books prepared. The Committee believes that it can best serve your interests if it can get from you an expression of your wishes in this matter. Will you cooperate with us? Postal regulations do not permit us to print a questionnaire in these pages and request that you fill it out, tear out the page, and return it to us. We request you therefore to consider the following questions and write to some member of the Planning Committee giving in writing your suggestions for yearbooks that you think would be valuable to the mathematics teachers of the United States. You need not copy the questions in the list, but it would be helpful if you would number your comments to correspond to the numbers of the questions. Will you do this soon?

Members of the Yearbook Planning Committee: Veryl Schult, Wilson Teachers College, Washington, D. C.; F. Lynwood Wren, Peabody Teachers College, Nashville, Tenn.; Walter H. Carnahan, Chairman, Purdue University, Lafayette, Ind.

Questionnaire

Subject: Yearbook planning for National Council of Teachers of Mathematics.

1. Suggest any problem connected with mathematics organization and teaching that you think would be suitable for treatment in a yearbook.
2. Would it be well to have a yearbook dealing with the organization and teaching of mathematics in the upper years of high school and in junior college?
3. Would curriculum planning in mathematics be a suitable subject for a yearbook?
4. Would a yearbook dealing with meaningful teaching of mathematics be useful?
5. Would it be well to plan a yearbook to deal with the problem of what mathematics to teach high school pupils who do not take the sequential courses?
6. What do you think of the possible value of a yearbook dealing with the evaluation of mathematics and mathematics teaching?
7. Would a yearbook dealing with topics useful in the enrichment of mathematics teaching be useful?
8. Is there need for a yearbook dealing with computation with approximate numbers?
9. Should there be a new yearbook dealing with any subject previously treated in a yearbook? Which yearbook?
10. Consider the topics suggested in 1 through 9 above; indicate your first choice for a yearbook.
11. Give the numbers of three yearbooks of National Council which you have in your personal library.

KNOW YOUR NATIONAL COUNCIL REPRESENTATIVES

By Kenneth E. Brown

Chairman of State Representatives, Department of Mathematics, University of Tennessee, Knoxville, Tennessee

Mississippi Representative

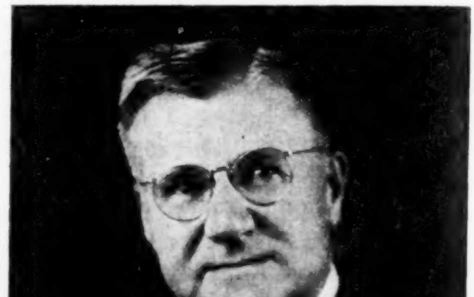
Miss Virginia Felder was born in Magnolia, Mississippi, and received her first teaching experience at Quitman High School. After gaining experience in teaching in both the high school and junior college, she joined the faculty of the Mississippi Southern College in 1948. She has been a member of the National Council of Teachers of Mathematics since accepting her first teaching position, and the

Virginia Felder

students in her methods class often hear her point out the value of membership in this organization. Her present address is Station A, Hattiesburg, Mississippi.

Florida Representative

F. W. Kokomoor, Department of Mathematics, University of Florida, Gainesville, Florida, is a native of Indiana but has taught in Florida since 1927. He was the first National Council Representative appointed in Florida and it is due to his continuous efforts that the number of National Council members in



Dr. F. W. Kokomoor

Florida has increased. Professor Kokomoor's interest in mathematics in general education is indicated by his writing the book *Mathematics in Human Affairs.* Dr. Kokomoor helped organize the Florida Council of Teachers of Mathematics which is now one of our active state organizations of teachers of mathematics. Under his leadership, a special National Council publicity campaign is being planned for this Fall which will culminate in the Eleventh Annual Christmas Meeting of the National Council of Teachers of Mathematics at Gainesville, Florida. Dr. Kokomoor extends an invitation to all mathematics teachers to attend this Christmas Meeting on the University of Florida Campus.

The National Council and Its Affiliated Groups

John R. Mayor, *Chairman*
Committee on Affiliated Groups

Action of The First Delegate Assembly of The National Council of Teachers of Mathematics, held in Chicago April 13–15, 1950, and subsequent endorsement by the Board of Directors, determined a pattern for relationships between the Affiliated Groups and the National Council. These Chicago deliberations and decisions of the thirty-nine official delegates are an important step forward in the coordination of the work of national, state, and local organizations devoted to the improvement of mathematics teaching at all levels of instruction. It is a step which is considered by all a fine beginning.

Of special importance was the recommendation of the delegates that the Affiliated Groups enjoy the privilege of having a part in the affairs of the National Council through such a plan as the Delegate Assembly which should meet each year at the time of the annual convention (Spring meeting) of the National Council, with each Affiliated Group represented by one delegate. In keeping with this recommendation the President and the Board of Directors have already set in motion plans for the Second Delegate Assembly to be held in Pittsburgh, March 29–31, 1951.

It was also recommended by the delegates that an Affiliated Group, representing an entire state, should elect the State Representative of the National Council for that state.

Requirements for Affiliation

The First Delegate Assembly voted that it be required that the

1. Group should have a written constitution and by-laws.
2. Group should maintain a list of its members with teaching positions and addresses and with indication of members of the National Council, and should file a copy of the list with the Chairman of Affiliated Groups.

The delegates approved as responsibilities of group affiliation the following points:

3. Annual renewal of affiliation should be required.
4. Renewal of affiliation and payment of dues should be made each year between October 1 and December 31.
5. An Affiliated Group should pay annual dues to the National Council under the plan

   51 to 150 members \$3.00
   151 to 250 members \$4.00
   251 to 350 members \$5.00
   over 350 members \$6.00

   with the condition that annual dues be waived for groups with 75% or more of their members also members of the National Council.

Desirable Eligibility Conditions

The majority of delegates voting also determined as desirable for eligibility for affiliation the following:

On Membership of the Affiliated Groups:

1. The minimum number of members in a group should be 50.
2. The membership should include teachers of mathematics from elementary school through college.
3. The membership should be open to all others interested in promoting the interests of the group.
4. At least 50% of the membership of the group should be teachers of mathematics.
5. The membership should be scattered over the area served by the group.

On Organization:

1. The group should provide for at least two regular meetings a year.
2. The group should provide for a coordinating committee to make possible the cooperation with other groups and with the National Council.
3. Groups should provide for continuity of leadership.
4. Officers should be members of the National Council of Teachers of Mathematics.
5. The purposes of the group should not be in conflict with purposes of the National Council of Teachers of Mathematics.

6. The group should make a special effort to help the elementary teacher.
7. The group should publish a bulletin, newsletter, or journal at least twice a year, and exchange these with other groups.

On Responsibility to the National Council:

1. The group should advertise the National Council, its programs, its publications, and should solicit members for the National Council and should make a definite effort to sell its publications.
2. The group should make an annual report to the National Council.

Obligations and Services of the National Council

The following obligations of the National Council to Affiliated Groups were recognized by the delegates meeting in Chicago:

1. The National Council should keep its members informed about matters of affilition through The Mathematics Teacher.
2. The Committee on Affiliated Groups should keep Groups informed about the activities of other Groups through a newsletter sent to Groups frequently.
3. The National Council should devote a portion of every convention to the work of Affiliated Groups.
4. The National Council should make it possible to have a representative of the Council, probably a member of the Committee on Affiliated Groups, to visit Groups and give help when desired.

It was recommended by the First Delegate Assembly that the National Council should:

1. Provide a speakers bureau.
2. Make available, free or at a slight cost, booklists, kits, and other teaching aids.
3. Conduct, with possible Group participation, studies and surveys of national scope.
4. Sponsor a traveling exhibit of textbooks, testing materials, posters, etc.
5. Provide direct aid to Groups in
   a. Organizing and conducting regional meetings whenever they are desired.
   b. Arranging and conducting workshops or institutes.
   c. Getting worthwhile materials published.
   d. Providing special help to elementary teachers.

Harry W. Charlesworth was chairman of the historic First Delegate Assembly and the official delegates were:

1. Association of Teachers of Mathematics in New England—Jackson B. Adkins
2. Association of Mathematics Teachers of New Jersey—Mary C. Rogers
3. Association of Teachers of Mathematics of New York City—Barnett Rich
4. California Mathematics Council—Dale Carpenter
5. Colorado Council of Teachers of Mathematics—Forest N. Fisch
6. Dade County Association of Mathematics Teachers (Florida)—Martha Rose Sanders
7. Dallas Elementary Mathematics Association—Mrs. Lorena Holder
8. Florida Council of Teachers of Mathematics—William A. Gager
9. Greater Cleveland Mathematics Club—Ona Kraft
10. Hillsborough County Council of Mathematics Teachers (Florida)—Mrs. Irma C. Ellis
11. Illinois Council of Teachers of Mathematics—Reuben A. Baumgartner
12. Indiana Council of Teachers of Mathematics—K. Eileen Beckett
13. Iowa Association of Mathematics Teachers—E. Glenadine Gibb
14. Kansas Association of Teachers of Mathematics—Gilbert Ulmer
15. Kentucky Council of Mathematics Teachers—Edith Wood
16. Louisiana-Mississippi Branch of The National Council of Teachers of Mathematics—Houston T. Karnes
17. Mathematics Section, Eastern Division, Colorado Education Association—Rose Myrtle Humiston
18. Mathematics Section, East Tennessee Education Association—Kenneth E. Brown
19. Mathematics Section, Georgia Education Association—Bess Patton
20. Mathematics Section of Maryland State Teachers Association—Margaret L. Heinzerling
21. Mathematics Section, Tennessee Education Association—F. Lynwood Wren
22. Mathematics Section, Texas State Teachers Association—Ida May Bernhard
23. Mathematics Section, Virginia Education Association—Allene Archer
24. Men's Mathematics Club of Chicago and Metropolitan Area—Walter Barczewski
25. Minnesota Council of Teachers of Mathematics—Emil Berger
26. Nassau County Mathematics Teachers Association (New York)—Alice M. Reeve
27. Nebraska Section of The National Council of Teachers of Mathematics—Theodora Nelson
28. Oklahoma Council of Teachers of

Mathematics—Mrs. Virginia C. Shike
29. Ontario Association of Teachers of Mathematics and Physics—Robert E. K. Rourke
30. Suffolk County Mathematics Teachers Association—Elaine Rapp
31. The Detroit Mathematics Club—Franklin Frey
32. Tulsa Mathematics Council—Muriel Lackey
33. Washington, D. C. Mathematics Club—Elinor Douglas
34. Washington, D. C. Benj. Banneker Club—Mrs. Ethel Harris Grubbs
35. Western Pennsylvania Mathematics Teachers Association—Catherine A. V. Lyons
36. West Virginia Council of Mathematics Teachers—Julia E. Adkins
37. Wichita Mathematics Association—Kenneth Nickel
38. Wisconsin Mathematics Council—Margaret A. Striegl
39. Women's Mathematics Club of Chicago and Vicinity—Virginia Terhune

MATHEMATICAL RECREATIONS

Edited by Aaron Bakst
School of Education, New York University, New York, N. Y.

How many times have teachers complained that pupils neglect the fundamental rules of operations? Usually the major complaint centers around the incorrect procedures of "cancelling" in the case of fractions. For example,

$$\frac{a+b}{c+b}$$

is cancelled so that the fraction a/c is obtained. We all know that such "cancelling" is wrong and it should be discouraged. But how can we explain the following?

$\frac{26}{65}$ is equal to $\frac{2}{5}$, and this may be obtained by "cancelling" the sixes in the numerator and the denominater.

$\frac{16}{64}$ is equal to $\frac{1}{4}$, and this may be obtained by "cancelling" the sixes in the numerator and the denominator.

What is wrong with these two examples? Are there any other cases of this type? Perhaps the readers of The Mathematics Teacher know of some. This department will be glad to publish an explanation if such is submitted. And how shall the pupils be discouraged from engaging in such "cancelling"?

Here is another case.

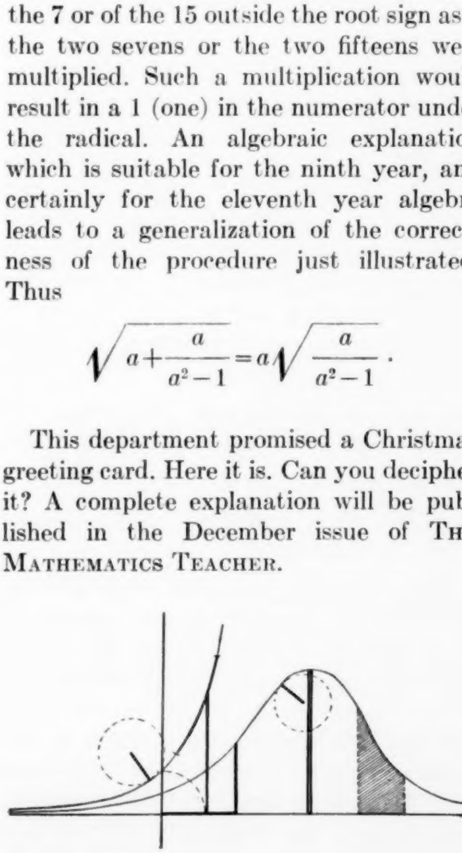
$$\sqrt{7\frac{7}{48}}=7\sqrt{\frac{7}{48}},$$

$$\sqrt{15\frac{15}{224}}=15\sqrt{\frac{15}{224}}.$$

We certainly cannot explain this taking of the 7 or of the 15 outside the root sign as if the two sevens or the two fifteens were multiplied. Such a multiplication would result in a 1 (one) in the numerator under the radical. An algebraic explanation which is suitable for the ninth year, and certainly for the eleventh year algebra leads to a generalization of the correctness of the procedure just illustrated. Thus

$$\sqrt{a+\frac{a}{a^2-1}}=a\sqrt{\frac{a}{a^2-1}}.$$

This department promised a Christmas greeting card. Here it is. Can you decipher it? A complete explanation will be published in the December issue of The Mathematics Teacher.



APPLICATIONS

By Sheldon S. Myers
University School, Ohio State University
Columbus, Ohio

Remember that this department is yours—yours to develop through your criticisms and suggestions, and, of course, contributions. Please do not hesitate to send us your ideas so that The Mathematics Teacher will have a department on applications which will be of maximum service.

We shall run once again our system of classification and numbering the applications with a minor change over the first issue: Ar. 1 Gr. 6–8 for Arithmetic, first application, grades 6–8; Al. 3 Gr. 9–10 for Algebra, third application, grades 9–10; C. Al. for College Algebra; P. G. for Plane Geometry; S. G. for Solid Geometry; T. for Trigonometry; P. for Proof; A. G. for Analytic Geometry.

Early in the fall, high school juniors and seniors taking physics are confronted with problems on the properties of matter involving units of density, weight, and volume. Mathematics teachers would render a service were they to help students clarify the basic mathematical ideas underlying these problems. Two basic ideas stand out in these problems: the simple formula $D=W/V$, and conversion of units. (D = density, W = weight, V = volume). Here is a problem developed by the writer to test mastery of these ideas:

Al. 2 Gr. 10–12 Dimensional Relationships in Physics

Imagine a new chemical compound by the name of "karin." It was discovered in a small European country where the "eel" is the unit of length. In this country the "groggin" is the weight of one cubic eel of water. The density of karin in this country's units is 3.4 groggins per cubic eel. Two other units of weight in this country are the "caliaca," 49.5 units of which equals one groggin, and the "quatina," one unit of which equals 26.3 groggins. Can you answer the following questions which arose in the course of marketing this new compound? (Solutions next month.)

1. How many cubic eels are there in 20.4 groggins of karin?
2. How many groggins would 50 cubic eels of karin weigh?
3. How many caliacas does 35 cubic eels of karin weigh?
4. How many quatinas does 85 cubic eels of karin weigh?
5. How many cubic eels are there in $3\frac{1}{2}$ quatinas of karin?
6. How many cubic eels are there in 99 caliacas of karin?

Ar. 3 Gr. 7–8 Lifetime of Some Animals

The possibilities of taking the ordinary drill materials of junior high arithmetic and weaving them around themes close to the interests of boys and girls are unlimited. Here are a series of problems developed at the University School, Ohio State University, in 1943.

So far, scientists do not know why some forms of animal life live so long while others live such a short time. The whale may live as long as 500 years. The elephant gets to be as much as a third as old as a 500 year old whale or _____ years. A turtle may live to be 7/10 as old as the old whale or _____ years. Some birds also live to a good old age. The swan reaches an age as old as the elephant or _____ years; the golden eagle has been known to live more than a century by 4 years or _____ years. A sparrow lives to be 40, the goose twice as old or _____ years; and the parrot $2\frac{1}{2}$ times as old as the sparrow or _____ years. Bears attain an age of 50 years, while the lion's life is about 15 years less than the bear's or _____ years. The house cat, of the same family as the

lion, sometimes lives to be 5/7 as old as the lion or ______ years better than a dog which lives to 15 years. Rabbits live to be 10 years old; mice two-thirds as old as rabbits or ______ years; squirrels live as long as mice; hogs live to be twice as old as rabbits; the toads four times as old as rabbits. Hogs live to ______ years, while toads live to ______ years.

Here are solutions to two of last month's problems.

Ar. 2 Gr. 9–12 Aliquoting

The reasoning involved in the solution of this problem is well within the abilities of high school students. 75 ml. out of 100 ml. would give 3/4 of the original sample. 3/4 are placed in 300 ml. and 100 taken. This would give 1/3 of the 3/4, or 1/4 of the original sample. The 1/4 is placed in 200 ml. and 10 ml. taken. This is 1/20 of 1/4, or 1/80 of the original sample of 10 grams. Thus, 1/8 gram, or .125 grams would be left in the 10 ml. of the final solution. By ratios, the calculation would look like this:

$$10\times\frac{75}{100}\times\frac{100}{300}\times\frac{10}{200}$$

$$=\frac{1}{8}\text{ gram or .125 grams}$$

T. 1 Gr. 11–12 Measuring the Earth's Circumference

Data:

Estimated north-south distance Kalamazoo to Cincinnati 240 miles

Length of pole = 82.62 inches

Length of Kalamazoo shadow = 53.99 inches

Length of Cincinnati shadow = 49.80 inches

See diagram for following solution:

tan of $\angle X$ at Kalamazoo

$$=\frac{53.99}{82.62}=.6898 \qquad X=34°36'$$

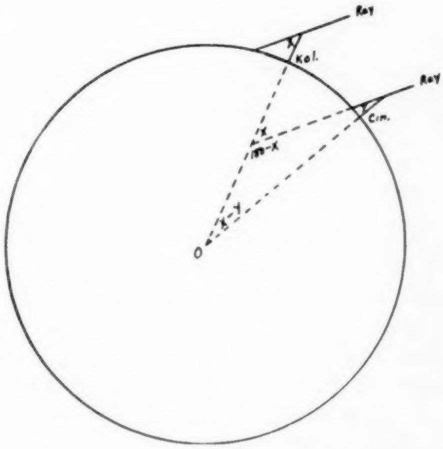
tan of $\angle Y$ at Cincinnati

$$=\frac{49.80}{82.62}=.6028 \qquad \angle Y=31°5'$$

$$\angle O=\angle X-\angle Y=3°30' \text{ or } 3.5°$$

$$\text{Circumference}=C=\frac{360\times 240}{3.5}$$

$$=24{,}700 \text{ miles}$$



P. 1 Gr. 10–12 A Problem for Thinkers

In a very well known university the names of the president, a professor, an instructor, and a janitor are Mr. James, Mr. Jones, Mr. Haines, and Mr. Ross, but not respectively. In the same university there are four students with the same names which we shall designate as James, Jones, Haines, and Ross. The following facts are known about these people:

1. The student with the same name as the professor belongs to the fraternity of which Ross is a member.
2. The daughter-in-law of Mr. Jones lives in Philadelphia.
3. The oldest son of the president is seven.
4. The wife of the janitor has never seen Mr. Ross.
5. The father of one of the students always confuses Haines with Jones in class but is not absent-minded.
6. Mr. Haines is the father-in-law of the instructor and has no grandchildren.

From these data it is possible to determine the respective names of the president, the professor, the instructor, and the janitor. Can you?

DEVICES FOR A MATHEMATICS LABORATORY

Edited by Emil J. Berger
*Monroe High School, St. Paul, Minnesota**

This section is being published as an avenue through which teachers of mathematics can share favorite learning aids. Readers are invited to send in descriptions and drawings of devices which they have found particularly helpful in their teaching experience. Send all communications concerning Mathematics Laboratory Projects to Emil J. Berger, Monroe High School, 810 Palace Avenue, St. Paul, Minnesota.

A PARALLELOGRAM DEVICE

In this article is described a simply constructed device for teaching the concept of the parallelogram and related principles. The device consists of two equal bars, AB and CD, joined to two other equal bars, AD and BC. (See Figure 1.) Convenient lengths are 12 inches and 8 inches for the first pair and second pair respectively. The joints are movable. They are made by drilling holes at A, B, C, and D, inserting eyelets, and then rounding them over. An advantage of this type of construction is that a hole is left at each pivot. Holes are also drilled at the midpoints E, F, G, and H of the four bars.

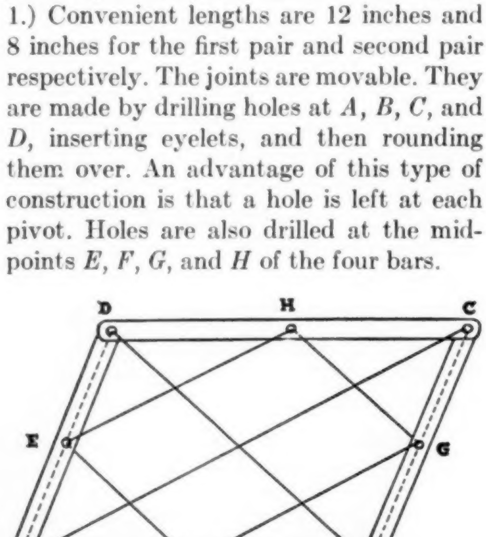


Fig. 1

Since the opposite sides of this quadrilateral are equal, $ABCD$ is a parallelogram. A student who draws static figures and studies their properties is apt to think of his proof as belonging only to a particular figure. If he can see manipulated a dynamic parallelogram which can be changed in shape and position, it may be easier for him to realize that the properties of a parallelogram are independent of its shape, size, and position. Thus he can see that no matter how the model is deformed, the opposite angles of the parallelogram appear to be equal. If the bar AD is moved so as to make angle DAB a right angle, then the resulting figure is a rectangle. Thus the definition of the rectangle as a special case of the parallelogram is illustrated. If the parallelogram is collapsed by moving sides AB and DC closer together, then the distance between them (the altitude) and the area both decrease. Thus the student can see that the area depends upon the base and altitude, and not upon the base and one side, as many students are prone to think.

Now let a piece of round, light weight, elastic braid be threaded through the holes as follows: up through A, down through C, up through B, and down through D. Then draw the elastic tight enough so that it will be under slight tension, and tie the ends together under the bar AD. When the parallelogram is deformed, the elastic can slide through the holes freely and so keep the two diagonals taut. This addition to the device makes it possible to illustrate the following two theorems: (1) A diagonal of a parallelogram divides the parallelogram into two congruent triangles; and (2) The diagonals of a parallelogram bisect each other. If a rigid bar such as a ruler is held along one of the diagonals the student will be able to see that although a quadrilateral is deformable, a triangle is rigid.

The elastic diagonals also make the model useful in explaining the vector

* *Acknowledgement:* Drawings for publication in this section were produced by Patricia McGroder, Macalester College, St. Paul, Minnesota.

parallelogram. If two adjacent sides are defined as vectors, then their resultant is the diagonal joining their common point. It is easy to see that when the angle between the vectors decreases, the resultant increases, and when the angle becomes 0° or 180°, the resultant is the algebraic sum of the vectors.

By threading a second elastic braid through the holes E, F, G, and H, and tying the ends between E and H the device becomes useful for illustrating two additional theorems. These are: (1) The line segments joining the midpoints of the sides of a parallelogram form a parallelogram whose area is one-half that of the original parallelogram; and (2) The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to one-half of it.

If bars AB and DC are taken as straightedges, then the device becomes a parallel ruler. Mount one bar against a table or wall and we have an adjustable bracket which will, for example, keep a shelf in a variety of positions which are always horizontal.

Next, let us remove the elastics, collapse the parallelogram, and pass one of the pivots across the adjacent long bar. (See Figure 2.) Now we have a geometric figure which is new to most students. It is a crossed parallelogram (often called a contra-parallelogram). In this new figure the diagonals AC and BD do not intersect because they are parallel. We can

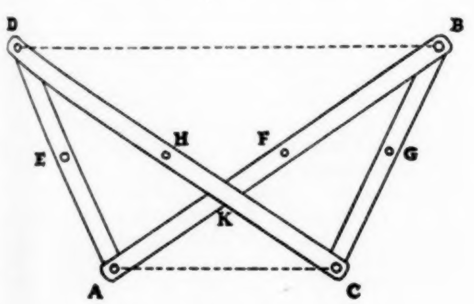


FIG. 2

easily show that triangles ADC and CBA are congruent, and in turn prove that triangles ADK and CBK are congruent. Thus, since the contra-parallelogram divides into triangles, we can prove a number of theorems about it based upon the theorems for triangles. We can establish criteria for similar contra-parallelograms, congruent contra-parallelograms, etc. Perhaps the most surprising fact about the contra-parallelogram comes from a consideration of the midpoints E, H, F, and G. It is easy to prove that these points lie on a straight line. So for every position of the contra-parallelogram the product of the distances EH and EF is a constant. Because of this remarkable property, if E is fixed to a plane and H is made to follow a circle passing through E, the locus of F must, by the geometry of inversion, be a straight line. Much more material on the contra-parallelogram and its use in solving the straight-line problem, trisecting the angle, and the like, can be found in the literature on linkages.

The above ideas are suggestive. They do not exhaust the purposes for which this simple device can be used. The device can be made of any durable material from which plane strips can be cut, such as heavy cardboard, fiber, sheet metal, sheet plastic, or wood. Making the joints with eyelets is convenient for threading the elastic, but small nuts and bolts or rivets could be used. In this case a small loop of wire through which to run the elastic could be fitted at each pivot.

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MATHEMATICAL MISCELLANEA

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6. *Rational Sets of Pythagorean Numbers*

Perhaps there are teachers who, in making up problems concerning right triangles, would like combinations of rational numbers for the three sides of a right triangle other than those that are so commonly used. Combinations of numbers commonly used are 3, 4, 5; 5, 12, 13; and 7, 24, 25.

Notice that in each combination just mentioned the square of the smallest number equals the sum of the other two and that the two larger numbers are consecutive numbers i.e.: $3^2=4+5$; $5^2=12+13$; $7^2=24+25$.

Applying this principle and starting with 9 as the smallest of the three sides of the right triangle, then $9^2=81$, and the two consecutive numbers whose sum is 81, are 40 and 41. Thus 9, 40, 41 are a combination that satisfies the Pythagorean principle of $a^2+b^2=c^2$.

If one starts with an even number such as 4, then $4^2=16$, and the two numbers with a difference of 1 whose sum is 16 are $7\frac{1}{2}$ and $8\frac{1}{2}$. This combination $(4, 7\frac{1}{2}, 8\frac{1}{2})$ will satisfy the Pythagorean principle for $4^2+(7\frac{1}{2})^2=(8\frac{1}{2})^2$. If one wishes to avoid fractions, he can double each number and the combination becomes 8, 15, 17.

The proof and explanation of this method used in obtaining sets of rational Pythagorean numbers follows:

(1) The Pythagorean principle of $a^2+b^2=c^2$ may be expressed as $c^2-b^2=a^2$. From this formula we see that the difference between the squares of two numbers must be a square if we are to deal only with rational numbers.

(2) The difference between the squares of any two consecutive numbers equals the sum of those two consecutive numbers. If n and $(n+1)$ are the consecutive numbers, their sum is $2n+1$. The difference of their squares is $(n+1)^2-n^2=n^2+2n+1-n^2=2n+1$.

(3) It follows from (2) that the difference between the squares of any two consecutive numbers will be a square if the sum of the two consecutive numbers is a square.

The first paragraph tells us that we must find two numbers the difference of whose squares is a square, and the next two paragraphs develop the fact that the difference of the squares of two consecutive numbers is always a square when the two consecutive numbers add up to a square. So our problem is to find two consecutive numbers whose sum is a square.

We can start with any number we wish as the smallest of the three Pythagorean numbers. The square of this number is considered as the difference of the squares of two consecutive numbers, or, which we have shown to be the same thing, the sum of the two consecutive numbers. We then divide this square into two consecutive numbers which become the second and third numbers of the Pythagorean trio.[1]

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[1] *Editor's note:* Not only teachers but also the better students will relish Mr. Cohen's simple and ingenious procedure. Students will also be interested in its historical background. The formula attributed to Plato may be obtained by letting p represent Mr. Cohen's smallest number, whence $p^2=n+(n+1)$, and then: $n=(p^2-1)/2$, $n+1=(p^2+1)/2$. We then have $p^2+(p^2-1)^2/2^2=(p^2+1)^2/2^2$ which when multiplied through by 4 gives the Platonic formula $(2p)^2+(p^2-1)^2=(p^2+1)^2$. The two larger numbers determined by Mr. Cohen's procedure always differ by 1 or 2. For a more general formula and a discussion of the early history of Pythagorean numbers see The Mathematics Teacher, Vol. XLIII (April, 1950), p. 102.

7. *A "Raid" on the General Conic*

This is not an attempt at an analysis of the general equation of the second degree in two variables. It is just a raid to get a few things which are easily picked up.

Suppose that the real conic

$$Ax^2+Bxy+Cy^2+Dy+Ey+F=0$$

has eccentricity e, has (x_1, y_1) as focus, and has $x\cos\alpha+y\sin\alpha-p=0$ as its associated directrix. The reader is asked to admit on the basis of his knowledge of the focus-directrix-eccentricity definition of a conic that there exists a real number k such that $k(Ax^2+\ldots+F)$ is identically equal to

$$(x-x_1)^2+(y-y_1)^2 - e^2(x\cos\alpha+y\sin\alpha-p)^2.$$

Then we would have by equating coefficients of like powers

$$kA=1-e^2\cos^2\alpha, \tag{1}$$

$$kB=-2e^2\cos\alpha\sin\alpha \tag{2}$$

$$kC=1-e^2\sin^2\alpha, \tag{3}$$

and three other equations involving x_1, y_1, and p. It is easy to show that (1), (2), and (3) imply:

$$k(A+C)=(2-e^2), \tag{4}$$

$$k^2(B^2-4AC)=4(e^2-1), \tag{5}$$

$$k^2\{(A-C)^2+B^2\}=e^4 \tag{6}$$

$$B=(A-C)\tan 2\alpha. \tag{7}$$

Equations (4), (5), (6), (7) in turn evoke the following observations: When $A=-C$ we have $e^2=2$ and the curve is a rectangular hyperbola. (B^2-4AC) has the same sign as (e^2-1) and may, as a consequence, be used as an "indicator" of the nature of the conic. Select $e=0$ and the conic is a circle for it must then be that $A=C$ and $B=0$. Finally the equation (7), in the more convenient form,

$$B\tan^2\alpha+2(A-C)\tan\alpha-B=0,$$

gives the slope of an axis of the conic. Elimination of k leads to a quadratic equation for e^2 and inspection of the equation leads to the property $(e_1)^{-2}+(e_2)^{-2}=1$.

To start with A, B, C, D, E, F and then to solve for $k, e, \alpha, x_1, y_1, p$ is not a heartbreaking task but it is not the most satisfactory way of reducing the general equation of the second degree.

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8. *A Practical Method for Raising Any Number to an Integral Power*[2]

The method described in this article will show a practical application of an algebraic formula to the solution of any problem wherein it is required to raise any number to an integral power. The following advantages are evident in the solution of such a problem: speed, accuracy, simplicity of calculation, and no mathematical tables required. From the evident simplicity of the basic formula and, in as much as the calculation reduces to the simplest operations, it would seem that this method would lend itself readily to high school instruction.

The basic formula for the solution of such a problem is:

$$x^2=(x+d)(x-d)+d^2$$

where x is the number of which the square is desired and d is an arbitrary number so selected as to make the solution simple. The number d is so selected that either $(x+d)$ or $(x-d)$ becomes an integer followed by zeros.

The solution can best be described by an example; Find $(784)^2$

$$\begin{aligned}(784)^2&=(784+16)(784-16)+16^2\\&=(800)(768)+256\\&=614{,}400+256\\&=614{,}656\end{aligned}$$

[2] *Editor's note:* This may be regarded as an extension of the procedures discussed in two articles, both titled "A Rule to Square Numbers Mentally" in *School Science and Mathematics*, vol. 14 (January, 1914) p. 71 and vol. 15 (January, 1915) pp. 20–21, by Robert C. Colwell and L. C. Karpinski respectively.

If the same problem is set down in the following manner, its computation is easily followed:

$x+d$	800
d	16
x	784
d	$16 \rightarrow d^2 = 256$
$x-d$	768
	800
$(x+d)(x-d)$	614,400
d^2	256
x^2	614,656

The full scope of this method lies in the fact that if d is large the basic operation is repeated on d until it has been reduced to a relatively simple number which can be squared mentally. To illustrate this feature we will use another example; the arrows in the problem indicate the succession of operations: Find $(7468)^2$

$x-d$	7,000	(note that here it is better to use $x-d$ than $x+d$)			
x	7,468		500		
d	$468 \rightarrow d$		468		30
$x+d$	7,936	d'	$32 \rightarrow d'$		32
	7,000				
$(x+d)(x-d)$	55,552,000		436	d''	$2 \rightarrow d''^2 = 4$
			500		
			218,000		34
					30
					1020
					4
			$1,024 \leftarrow d'^2$		1024
	$219,024 \leftarrow d^2$		219,024		
x^2	55,771,024				

Note that in the above illustration it was never necessary to multiply by more than one positive digit for any one operation.

For the higher exponents the following formulas are used. It should be noted that these are based on the basic formula.

$$x^3 = x^2\left[(x \pm d) \mp d\right] = x^2(x \pm d) \mp x^2 d$$

$$x^4 = (x^2)^2$$

$$x^5 = x^4\left[(x \pm d) \mp d\right] = x^4(x \pm d) \mp x^4 d$$

$$x^6 = (x^3)^2$$

. . . .

Additional formulas for higher exponents can easily be derived in a similar fashion. The $\pm$ and $\mp$ notation used within the brackets for the odd exponents indicates that if $x^2(x+d)$ is used then x^2d must be subtracted, whereas if $x^2(x-d)$ is used then x^2d must be added.

We will present here one further example to illustrate how a number is cubed. The remainder of the formulas will follow from the other examples already shown. Find $(757)^3$

(See next page for solution)

$x+d$	800		
x	757		40
d	$43 \rightarrow d$		43
$x-d$	714	d'	$3 \rightarrow d'^2=9$
	800		
$(x+d)(x-d)$	571,200		46
			40
			1840
			9
	$1,849 \leftarrow d^2$		1849
x^2	573,049		

The above x^2 was found with our basic formula, now; $x^3=x^2[(x+d)-d]=x^2(x+d)-x^2d$

x^2	573,049	x^2	573,049
$x+d$	800	d	43
	458,439,200		1 719 147
			22 921 96
(subtract)	$24,641,106 \leftarrow x^2d$		24,641,107
x^3	433,798,093		

The most difficult operation performed here was finding x^2d which involved multiplying by a two digit number. The value of d cannot be reduced in this case but it should be understood that for all values of x from 0 to 10,000 the maximum value that d can attain is 500.

The speed of solution of these problems depends mostly on the value which d takes for each problem. The examples used here were selected to illustrate the generality of the method. However, in many instances, even though the value of x is large, d will be small and consequently in these cases the solution will be even more rapid.

In order to give some idea of the speed that can be attained by this method, there are listed below some random examples of actual time trials.

$(78)^2=6084$	9 sec.
$(992)^2=984,064$	11 sec.
$(786)^2=617,796$	16 sec.
$(8.6)^2=73.96$	20 sec.
$(9.46)^2=89.4916$	44 sec.
$(8586)^2=73,719,396$	1 min. 10 sec.

$(84)^3=592,704$	50 sec.
$(728)^3=385,828,352$	2 min. 3 sec.
$(285)^4=6,597,500,625$	2 min. 42 sec.
$(86)^8=2,992,179,271,065,856$	5 min. 30 sec.

These actual tests show that with practice the user can compute problems of this nature with amazing speed and accuracy.

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9. *Transposition of Digits*

Bookkeepers and others working with numerical data need systematic procedures for discovering the location and nature of errors as well as for checking for their existence. One of the most common errors is that of transposing the digits of a number as it is being copied. In this case the amount of the error, i.e., the difference between the correct and entered values, is always divisible by 9. Hence, it is standard procedure for bookkeepers to divide by 9 the amount by which a trial balance fails to balance. If there is no remainder the bookkeeper then looks especially carefully for transposition errors as he checks his entries.

The proof for this property of transposition errors emphasizes the nature of our decimal system, and uses simple algebraic symbolism and factoring in connection with an interesting and common application of mathematics.

Let $a, b, c, \cdots, g, \cdots, q, r, s$, be the digits of a number written to the base ten. The number then is

$$a\times 10^n+b\times 10^{n-1}+c\times 10^{n-2}+\cdots$$
$$+g\times 10^{n-m}+\cdots+p\times 10^2+q\times 10+r.$$

Assume two digits say b and g to be interchanged. The new number is then

$$a\times 10^n+g\times 10^{n-1}+c\times 10^{n-2}+\cdots$$
$$+b\times 10^{n-m}+\cdots+p\times 10^2+q\times 10+r.$$

The difference between these two polynomials is then

$$(b-g)10^{n-1}+(g-b)\times 10^{n-m}$$

which simplifies to

$$(b-g)(10^{n-1}-10^{n-m})$$

or

$$(b-g)10^{n-m}(10^{m-1}-1).$$

But $(10^{m-1}-1)$ and hence the total amount of the error is easily seen to be divisible by 9.

Students may also be interested in noting that this type of error, for obvious reasons, will not be revealed by the "check by nines." Further, just as in a number system to a different base, b, the check by nines has as its counterpart a "check by $(b-1)$," so a transposition error in a number system with base or radix, b, will be divisible by $(b-1)$.

10. *One-Place Logarithms*

Large numbers scare pupils. Although Professors can talk with ease about quintillions and sextillions, any number with more than four digits is out of the comfortable range of an average high school pupil's comprehension.

Many mathematics pupils never learn to appreciate the use of logarithms as a time-saving device because they are introduced too soon to long tables filled with four and five digit figures.

I believe that by spending one day on simple one place logarithms, a teacher can double the number of pupils in the class who can really grasp all the fundamentals of this handy, but intricate, tool.

First explain that the logarithm of a number is the exponent which applied to 10 will produce the given number. Then put the following table on the blackboard.

(*See top of the next page*)

Next have the pupils learn to do simple problems using these numbers, as 8 times 25, or 16 times 32, or 1/25th of 10,000.

Log N	N	Log N	N	Log N	N	Log N	N
0.3	2	1.8	64	0.7	5	4.2	15625
0.6	4	2.1	128	1.4	25	4.9	78125
0.9	8	2.4	256	2.1	125	5.6	390625
1.2	16	2.7	512	2.8	625	6.3	1953125
1.5	32	3.0	1024	3.5	3125	7.0	9765625

Then explain the negative logarithms of fractions. Thus the log of $\frac{1}{2}$ or .5 is $9.7-10$. Explain why it is stated this way to make a positive mantissa, .7, instead of -0.3.

Some pupil is sure to note that though 125 doesn't equal 128, they have the same logs. Just explain that the tables are not exact, but will work in all general cases and are within $2\frac{1}{2}\%$ or 1/40 of perfect accuracy.

From this table the log of 98 is 2.0. The log of 49 must be 1.7 and the log of 7 therefore must be $0.8\frac{1}{2}$.

Now the log of 126 must be about 2.1. Then log of 63 is 1.8, and since the log of 7 is $0.8\frac{1}{2}$ the log of 9 is $0.9\frac{1}{2}$. The log of 3 must be $0.4\frac{3}{4}$.

The log of 99 is 2.0, hence the log of 11 would be $1.0\frac{1}{2}$.

Now have the pupils figure out the log of 44, 13, 1, 200, .065, etc. by the use of the table they have.

Pi is almost the same as the square root of 10, or 0.5. By its use the areas of circles can then be found.

With the use of 1-place logs, problems and quiz-kid stunts, such as the following, can be figured out without pencil and paper entirely in one's head:

What is the 12th power of 5? 12 times .7 is 8.4 or ten million times 25, or 250 million.

What is 2 to the 20th? A little over a million.

What is the probability of picking at random the winner of 19 out of 20 contests? This is $20\cdot(\frac{1}{2})^{20}$. Since 2^{20} is approximately 1,000,000, the probability is $20\cdot(1/1{,}000{,}000)$ or about 1 chance in 50,000.

What is the cube root of 10 million? 3 into 7.0 is $2.3\frac{1}{3}$. This gives a value of a little over 210 which has a log of $2.3\frac{1}{4}$.

What is 7 to the 7th power? $0.8\frac{1}{2}\times 7$ is $5.9\frac{1}{2}$ which gives a value very near 900,000 since the anti-log of 6.0 is a million.

What is the area of a circle with a 32 foot radius? 1.5 plus 1.5 plus 0.5 is 3.5 or about 3,200.

Some textbooks discuss natural logarithms and have such problems as: What's the log of 8, using 4 for a base? Such procedures merely confuse many of the pupils. Merely mention that there are natural logarithms and let it go at that. Emphasize the use of logarithms as a tool in everyday business and forget the applications that are used only in a few obscure calculus problems.

Logarithms were invented to be used, a tool to simplify man's work. After the pupils are adept in one-place logarithms, then the teacher can show the use of four- and five-place tables. In this way pupils are much less likely to be mixed up in a mass of figures than if they are introduced to the tables five minutes after being told what a logarithm is.

By devoting one day to one-place logarithms, I believe that students will start using logarithms in some of their science and bookkeeping courses (to figure triple discounts for example) rather than merely to absorb a bit of the theory because it is in the textbook and the teacher said it was important.

An idle tool is of no use to anybody.

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TOPICS OF INTEREST TO MATHEMATICS TEACHERS

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Arithmetic in the High School

Whether mathematics teachers like it or not, arithmetic in the high school, in one form or another, is here to stay. It is now nearly ten years since the National Council's *Committee on Arithmetic* in the 16th Yearbook somewhat timidly suggested extending arithmetic instruction into the high school. In the interim, a world war began and ended (?), revealing tragic deficiencies in the mathematical competency of the general public. At the same time, a revolution took place in the teaching of arithmetic in the elementary schools, the full force of which is not yet altogether clear, although it is exceedingly promising.

Much of the recent discussion concerning arithmetic in the high school has centered around diagnostic studies and remedial teaching. But the matter goes deeper and has broader implications. Instruction in arithmetic is intimately related to other significant problems: the fate of general mathematics; mathematics in the junior high school; the controversial ninth-year program; the double-track plan; mathematics and the core curriculum; consumer education; non-academic mathematics; and business arithmetic.

It is about time to stop fretting about remedial arithmetic in the high school as a necessary evil and a special headache. Let us be realistic and constructive. If the current emphasis on meaning theory and the recent trend in upward grade-placement of topics are both sound policies, and there seems to be every reason to believe that they are, then why not consider arithmetic in the high school as an integral part of an organic program? If this were done, "remedial work" could be replaced by continued developmental teaching, accompanied by the usual cumulative reteaching; also, more attention could be given both to general competency as well as to increased understanding of the quantitative nature of the society in which we live. Because of their greater maturity, high school pupils are far more likely to profit from instruction in the consumer, social, and community aspects of arithmetic than are 7th and 8th grade pupils. In the light of these observations, the following references may be of interest to classroom teachers and administrators alike.

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(*continued on p. 367*)

AIDS TO TEACHING

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BOOKLETS

B.35—Why Study Mathematics?

The Canadian Mathematical Congress, Engineering Building, McGill University, Montreal, Canada.

Booklet; $7'' \times 8\frac{1}{2}''$; 33 pages; Single copies—\$.50, twelve or more—\$.40 each. (1949?)

Description: A committee of eight, appointed by the Canadian Mathematical Congress, has prepared this booklet to help pupils to understand, teachers to motivate, and both to appreciate the uses of mathematics. Wisely, these uses are not restricted to practical, vocational ones, but are balanced with some more cultural ones. In fact, there are three chapters of each type: the first three deal with careers that need (1) an expert knowledge of mathematics, (2) a knowledge of special fields of mathematics, and (3) a substantial foundation of mathematical study; then there are three more chapters which deal with the mathematics in the education of a competent citizen, the contribution of mathematics to sound reasoning, and the intellectual and aesthetic satisfactions from mathematics. There are many line drawings, with color, which relieve the monotony of the printed page.

Appraisal: This is one of the most attractive booklets on mathematics now available. It is extremely important to have such information to inform and convince secondary school students of the future to be expected from their study of mathematics. This audience is constantly kept in mind; and the type of approach should be extremely successful. The interesting kinds of examples used, the lack of pedantic detail, and the fascinating pictures which combine humor and illustration in the spirit of modern art all result in a book which can be read rather than studied.

A comparison with the Guidance Report of the National Council is inevitable. This booklet is bigger, more attractive and more expensive. Also it is not trying to do the same job: the Guidance Report had an amount of detail in definite figures which impresses students all the more because such definite detail is often not available through their teachers; this Canadian booklet, by expanding the meaning of "use" beyond the vocational, has a larger topic to cover and therefore does so in less detail.

Every mathematics teacher must have a copy of this booklet and every student, too, if possible. Otherwise the school should have plenty of copies to lend to students.

B.36—Path of Flight

Superintendent of Documents, U. S. Government Printing Office, Washington, D. C.

Booklet; $7\frac{3}{4}'' \times 10\frac{1}{4}''$; 32 pages; Map enclosed; \$.40 each, 25% discount on 100 or more copies to same address.

Description: This is one of the pamphlets provided to private pilots by the Civil Aeronautics Administration, and the most

interesting to mathematics classes. Without being too technical, it covers the following topics: aids to navigation (including charts), measurement of direction (e.g. parallels and meridians, compass heading), basic calculations (e.g. speed, angles, course, heading, fuel consumption), chart reading, pilotage, the wind triangle (solved graphically), the radius of flight, and some special problems based on the vector solution of triangles. There are thirty diagrams and in a pocket at the back of the booklet one sheet of the world aeronautic chart for practice purposes.

Appraisal: The subjects taught in this interesting booklet are not a standard part of the secondary curriculum, and time will be found for them only at the expense of more traditional, abstract topics. This substitution will seem to many teachers to be detrimental, and yet may be easy in the second track courses where traditional subjects are not taught and new material is sought. Even in the sequential courses, new interest and appreciation of the topics which remain may stem from a proper substitution of some material of this type.

One of the most powerful features of this booklet is the fact that it is not written for mathematics classes, or for schools at all. Here is an efficient way for pilots to solve some of their urgent problems which cannot avoid the terms, concepts and methods of mathematics. Aviation is not as fascinating as it was once, but it is still not so commonplace that it will not pep up mathematics teaching with its applications.

B.37—School Savings Journal for Classroom Teachers, Spring 1950

Booklet; $11'' \times 13\frac{1}{2}''$; 12 pages; Free.

B.38—School Savings in Action

Booklet; $5\frac{3}{4}'' \times 9''$; 13 pages; Free.

B.39—Budgeting through School Savings

Booklet; $5\frac{3}{4}'' \times 9''$; 19 pages; Free.

B.40—Lessons in Arithmetic through School Savings

Booklet; $5\frac{3}{4}'' \times 9''$; 26 pages; Free.

B.37 through B.40 available from Education Section of the U.S. Savings Bonds Division, Treasury Department, Washington, D.C. or from the Treasury Bond office in each state (list of these will be found in each of the booklets listed here).

Description of B.37: This newspaper-like publication, with attractive, colored cover gives stories and descriptive articles of schools' experiences with bond programs. It is definitely planned to have something to appeal to all teachers from grades 1–12. The center pages make a poster, in color, $13\frac{1}{2}'' \times 22''$.

Description of B.38: This is a general introductory booklet describing what a school savings program is and suggesting the types of activities to be carried on in schools.

Description of B.39: Here is a helpful booklet which talks about the methods and advantages of budgeting in spending your money. A great deal of emphasis, too much in fact, is put on the place of government bonds in such a program.

Description of B.40: Lessons in Arithmetic is intended to give definite teaching suggestions: questions, exercises, problems, and activities to aid the teacher of grades 1–6 in propagandizing for government savings bonds. A similar booklet (B.21), for grades 7–12, was reviewed in this department in April, 1949.

Appraisal of B.37 through B.40: This is an excellent series of booklets which should not exist. Why should the government have the ability to put out booklets which distort and over-emphasize the position of savings bonds in arithmetic, budgeting and life in general? There are so many other applications of arithmetic which are just as deserving and just as important, but their propagandists have not the bottomless well of funds which the Federal Government has to create advertising. The purchases of food, clothing, housing, drugs, washing machines, automobiles and other commodities are just as worthy of discussion, but no such

booklets are available to compete with those of the government.

The booklets listed above are good, they are excellent, and it is too bad to refuse their use to any teacher, for the material in them is well worked out. But what teacher can avoid a one-sided presentation when so much usable material is available on one application of the subject of arithmetic. If the use of this material can be balanced with other items in the lives of the children, it should certainly be admitted.

This just serves to raise legitimate questions in any teacher's mind concerning the ability of a government to place biased teaching materials in schools which stress, to excess, the value of the activities of the government. Where will the line be drawn?

B.41—The Story of Figures

Burroughs Adding Machine Co., 6071 Second Boulevard, Detroit 32, Michigan.

Booklet; $5\frac{1}{2}'' \times 7\frac{1}{2}''$; 35 pages; Free on single copy basis.

Description: Ten pages are devoted to a brief history of numbers and the four fundamental processes, eight to the history of mechanical computation, and eight to the work of William Seward Burroughs in perfecting the first adding machine. The last seven pages are devoted to advertising. The story is simply written but is extremely interesting. It is obviously unbalanced in favor of the Burroughs Adding Machine Company, but the alert teacher can compensate for this misplaced emphasis.

The cover design consists of a series of concentric circles containing characters which illustrate man's methods of recording digits from the early Babylonian cuneiform numerals to our present number system. Twenty-four attractive line diagrams serve to make the booklet a well illustrated supplementary aid.

Appraisal: This booklet may be used to contribute toward the general objective of acquainting students with the rich mathematical history of those factors which led to eventual development of the Arabic decimal number system and the development of mechanical devices to simplify operations concerning numbers. It may also be used to bring about a greater understanding of such mechanical devices as the "pivot," "locked keyboard," and "automatic control," which underlie the operation of modern business machines and minimize errors due to careless depression of keys and to minor differences in operating technique.

The vocabulary used in the booklet is suitable for students of high school age. Its emphasis on machines, however, is unfortunate. (Reviewed by Bernard Singer, Hyannis, Massachusetts.)

EQUIPMENT

E.35—Flash Cards on a Wheel

L. M. Wright Co., 686 East Mariposa Street, Altadena, California.

Arithmetic device; 23 inches high, 17 inches wide. Wooden stand, \$3.50; pressboard stand, \$1.75. Set of five blank card-wheels \$1.50; five card-wheels printed with 100 multiplication and 90 division combinations, \$3.50; and ten card-wheels printed with all 390 basic combinations, \$7.00.

Description: The device consists of a wooden or press-board stand with circular part $16\frac{3}{4}$ inches in diameter. At the center top of this circle, a section approximately 6 inches high and 2 inches wide has been cut out. The oak tag card-wheel is attached to the back of the stand, and may be rotated so that one combination at a time shows through the opening. For instance, on one of the multiplication-division wheels, twenty multiplication combinations are on one side and related division combinations are on the reverse side. The figures are placed so that the top number of either the multiplication or the division combination is the answer

to the combination on the reverse side. For example, one multiplication combination is written $\frac{2}{3}$ and the opposing division combination is written $\frac{6}{3}$. The figures are $1\frac{1}{4}$ inches tall and can be seen at a distance.

Appraisal: This device is intended to replace flash cards. The claim is that it is less tiresome for the teacher to use, apparently because the stand holds the cards and the operation may be performed with one hand. Although some consideration has been given to the choice of combinations, a few are questionable. For example, 4 times 5 appears on the same card-wheel as the 1's, 2's, and 3's. This is unnecessary, as there are some omissions in the groups mentioned. A teacher would prefer using the card-wheel after certain understandings have been well established with meaningful practice, so that content should include relevant learnings and exclude all irrelevant ones.

Another criticism concerns the division form used. The division is printed as an improper fraction, a form which is not familiar to children who are learning the division combinations.

The device may have merit and serve a purpose in some situations, but it is advised that the blank cards be purchased and teachers stamp the numbers in accordance with practice in their communities. (Reviewed by Katherine Murphy, Newton, Massachusetts.)

FILMS

F.56—How to Judge Facts

Coronet Films, Coronet Building, Chicago 1, Illinois. Educational Collaborator, William G. Brink, Northwestern University, Evanston, Ill. 1948.

16 mm. film; Black and White ($45); Color ($90); 10 minutes.

Description: The film opens with a discussion between the editor of a school paper and one of his reporters. Reporter Jim has written an article announcing that the fund for new football uniforms is to be spent to buy a second projector for the school. Jim feels sure that he has stated the facts of the case in his story. Editor Bob is suspicious that Jim's "facts" may be only rumors and asks how they were gathered. The film then relates the "facts" as assembled from several students, from a theatre owner and from a mother. Not satisfied with these "facts," the editor suggests that Jim talk with the principal about the matter. Jim learns from him that the Parent-Teacher Association plans to buy the projector with its funds and that the football treasury has never officially been considered as a source of the money. The next section of the film deals with the errors in Jim's information. These include a false analogy, a false assumption, a platitude, and a lack of clear definition of terms.

Appraisal: The film has merit in that it emphasizes the difference between opinion and fact. Certain common types of errors in reasoning are specifically illustrated in situations within the immediate experience of children. To some of the pupils, certain bits of the story seemed exaggerated. Others felt that the same difficulty would have been encountered in any single story which tried to illustrate all these points. Possibly a more realistic treatment of each error type could have been achieved with isolated illustrations. On the other hand, if that were done the film would lose a certain continuity which it now has. It would be helpful with geometry students *if* considerable time has previously been given to a study of these types of errors in reasoning. In other mathematics classes, its value would be more general than specific. It is, of course, the function of all subject fields to teach for clear thinking. This film is an aid to such an objective whether it be utilized in mathematics or in some other area. The level is junior and senior high school. The words of the student actors are at times indistinct, but those of the commentator are clear. (Reviewed by Frances Burns, Oneida High School, Oneida, New York.)

FILMSTRIPS

FS.73—Areas of Triangles—Areas of Trapezoids

Photo and Sound Productions, 116 Natoma Street, San Francisco 5, California.

Educational collaborator, O. W. McGuire, 35 mm. filmstrip; Black and White; 31 frames.

Description: This is Part V of the Study of Measurement series. It assumes that pupils are familiar with the content of Part IV which deals with the area of a rectangle and of a parallelogram. The first section of the filmstrip is concerned with the area of a right triangle. In the opening frame, *triangle* is defined and pictured but only one illustration (a right triangle) is used. The next frame shows a rectangle with sides a and b, recalls that its area is $a\cdot b$ and states the formula $A=ab$. The rectangle is then divided into two right triangles by drawing a diagonal; the equality of these triangles is mentioned; the area of either triangle is given as $\frac{1}{2}$ of $a\cdot b$; translation to the form $A=ab/2$ is made; the fact that $ab/2$ means "ab divided by 2" is stressed. The second section defines and illustrates *base, vertex* and *altitude* of a triangle. It then relates the base and altitude of the right triangle to the length and width of the rectangle and extends the triangle area rule to the vocabulary "one half the base times the altitude." By association with the parallelogram, the method is then shown to apply also to the general triangle. The last section develops the formula for the area of a trapezoid. The figure is first defined and pictured. The trapezoid is then shown to be made up of two triangles which are not equal but which do have equal altitudes. The areas of the two triangles are given as $am/2$ and $an/2$. These are then added to get $A=a(m+n)/2$. A problem follows in which the area of the trapezoid as worked by this formula "checks" with the result obtained by finding the areas of the two triangles and adding them. Four frames at the end of the filmstrip are used for recall and for emphasis of the formulas already learned.

Appraisal: The level is junior high school since the treatment is entirely intuitive. In the frames which define base, altitude and vertex, the illustration used is a right triangle with the right angle at the vertex. A general triangle would have been a better choice. The use of both the capital and the lower case letter in $A=ab$ might be confusing to children of this age level. The addition of $am/2$ and $an/2$ to give $a(m+n)/2$ would, perhaps, put considerable strain on the pupil's available knowledge of literal mathematics. With these exceptions the content and vocabulary are satisfactory for junior high school students. It is probably true that the desired objective could be better accomplished with the usual equipment of the classroom than it could be done with this filmstrip. On the other hand, it might be helpful for briefing a pupil who has been absent, or for a slow pupil who needs additional attention, or to provide the spice of variety for reteaching or review. (Reviewed by Frances Burns, Oneida High School, Oneida, New York.)

INSTRUMENTS

I.26—Pocket Pelorus

Texaco Waterways Service, 135 East 42 Street, New York 17, N. Y.
Cardboard Pelorus; $5\frac{1}{2}''\times5\frac{1}{2}''$; Free.

Description: This light cardboard model opens to disclose on the right-hand page a four-inch pelorus composed of a compass rose and set of movable vanes mounted on a common axis with a lubber line marked on the mount. The compass rose is graduated in degrees and also in points of the compass. The vanes can be used to sight bearings of distant objects. On the left-hand page are simple directions for using the device to take bearings from a boat. Applications are also given to the following methods: bow and beam bearings, cross bearings, and two bearings and run between.

Copies are available to teachers either singly or in moderate quantity. For large classes, the members should write letters or post-cards to the address above for individual copies.

Appraisal: Here is an excellent, simple, inexpensive model to use at many levels of mathematics. It can illustrate angles, direction, parallel lines, angle measurement, use of charts, isosceles triangles, intersection of straight lines, similar triangles, and even trigonometric solutions of oblique triangles. Some classes will use some topics, other classes will want different ones. If the teacher who is awed by some of the words which, it is true, are more navigation than pure mathematics, would just plunge in, it would soon become obvious that the applications are both real and easy. How much more fascinating for students to work out problems from their own data, even though they have sailed around the school yard in imaginary boats!

MODELS

M.13—Straight Line Unit

Science Service and the National Council of Teachers of Mathematics. M. H. Ahrendt, Anderson College, Anderson, Indiana. Construction Kit; $5\frac{1}{2}'' \times 3\frac{1}{2}''$; 1950; \$.50 each, three for \$1.00.

Description: This small kit contains a heavy piece of cardboard with five holes which acts as a base for the models to be constructed; two pieces of cardboard of lighter weight which are stamped so that they can be easily broken into ten strips for arms of linkages and twelve cardboard washers; eleven paper fasteners; ten metal rivets; and an eight-page, illustrated booklet ($5'' \times 6''$) which describes the models to be constructed. The booklet shows how one can make the following linkages with the parts supplied: Watt's linkage; the Peaucellier cell; another, seven-bar, straight-line linkage; and a re-use of the Peaucellier cell to draw the cissoid curve. In addition, the booklet gives a short description of methods to construct more permanent linkages and ends with a bibliography of eight items.

Appraisal: It is certainly noteworthy that the Kit Committee of the National Council has succeeded again in getting so much good into so little space. It is unfortunate, however, that mass production due to mass appeal, or industrial subsidies, or some other means cannot reduce the cost of these kits to a smaller figure. For those who have never built linkages, this kit will serve as a quick, easy, direct and enjoyable introduction. The examples chosen are simple and important. If it were possible to supply enough material so that all linkages could have been constructed without dismantling those previously made, it would lead to a longer life of the whole set.

However, since this kit should serve to kindle the interest in the construction of linkages, and to show how one can construct them from odds and ends of cardboard, any teacher and class that once starts will no doubt continue making bigger, stronger, and more complicated ones for years to come.

Topics of Interest

(continued from p. 361)

Mergendahl, C. H. and Foster, B. R. *One Hundred Problems in Consumer Credit.* (Pollack Pamphlet, No. 35). Newton, Mass., Pollak Foundation for Economic Research, 1938. 55 p.

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Schorling, R., Clark, J. R., and Lankford, F. G. *Mathematics for the Consumer.* Yonkers, N. Y., World Book Co., 1947. 438 p.

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NOTES ON THE HISTORY OF MATHEMATICS

Edited by Vera Sanford
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COUNTERS; COMPUTING IF YOU CAN COUNT TO FIVE

Ordinary computation can be accomplished with a minimum of learning by using the loose counter abacus and the counting board. The counting board is a flat surface marked with a series of parallel lines whose values are 1, 10, 100, 1000. The counters are small, easily handled objects,—pebbles, metal disks about the size of a penny, or even, for present-day experimentation, paper clips. The position of a counter shows its value. The number 1432 is indicated by placing one counter on the thousands line, four on the hundreds line, three on the tens, and two on the units line.

Reckoning with counters has left its mark in such words as "calculate" and "calculus" from the Latin *calculus* (a small stone), the "counter" in a store which originally was a counting board, to "cast up accounts" from throwing the counters on the board, and the terms "carry" and "borrow" which described the actual process.

The origin of the counting board and much of its history are not known. Herodotus (*c.* 425 B.C.) notes that "In writing and in reckoning with pebbles, the Greeks move the hand from left to right, but the Egyptians from right to left." This indicates that the reader was familiar with the process and would be interested in the difference between the Greek practice and that of the Egyptians and it also indicates that the lines of the abacus were vertical. In both Greek and Latin literature, references to the counters appear in context such as the following,—"He (Solon) used to say that men who surrounded tyrants were like the pebbles used in calculations; for just as each pebble stood now for more, now for less, so the tyrants would treat each of their courtiers now as great and famous, now as of no account." We also know that the equipment of a Roman school boy included a bag of counters as well as a wax tablet. And there are still extant Roman abacuses of the type in which beads slide on rods or where studs move in grooves. Details as to the use of the abacus are lacking.

There are references to the use of counter casting in the fourteenth and fifteenth centuries, but there appears to be no actual description of its operation.

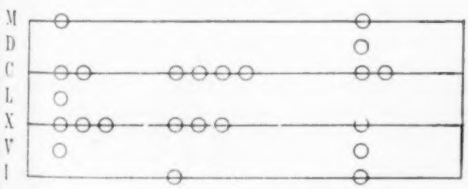
In the sixteenth century, a considerable number of arithmetics appearing in northern Europe, especially in Germany, included accounts of computation with the loose counter abacus which seems to have become a well established mercantile practice. German arithmetics were outstanding in this regard. In England, the earliest known book in English on arithmetic (1539) has the following descriptive title:

> AN INTRODUCTION
> for to lerne to recken with the pen, or with the counters accordynge to the trewe cast of Algorysme, in hole nombers or in broken/ newly corrected. And certayne notable and goodlye rules of false posytions therevnto added, not before sene in oure Englyshe tonge, by whiche all maner of difficyle questionyons may easily be dissolusd and assoylyd.

This was quickly followed by Robert Recorde's *Grovnd of Artes* (*c.* 1542) which had a section on the use of counters. The subject seemed to have been neglected in Italy and in France, the textbooks keeping to the arithmetic with the pen. Strangely enough, although the subject was omitted from the earlier editions of a popular arithmetic in France, a book first printed in 1656, a section on counters was introduced in the edition of 1705 and appeared in at least three editions including

that of 1781. It is a bit surprising that the later editors chose to put this topic in, but the explanation is as follows: "This arithmetic is quite as useful as that which is done with the pen since by counters, one can perform every calculation of which he has need in business. This way of computing is more practiced by women than by men; nevertheless many people who are employed in the Treasury and in all the government departments make use of this with great success."

The loose counter abacus of the sixteenth century differed from the Greek one in that the lines were marked from right to left instead of up and down. The line nearest to the computer had the lowest value. The counting board might be of stone with the lines cut on it. It might be of wood with lines drawn in chalk for temporary use or painted on permanently. It might be a table cloth with the lines embroidered on it. The lines had the values 1, 10, 100 etc., and the spaces between had the values 5, 50, 500. There is a close correspondence between these markings and Roman numerals. A number is indicated by placing counters on the lines and spaces as the case requires. The accompanying figure shows how the numbers 1285 and 431 would appear.



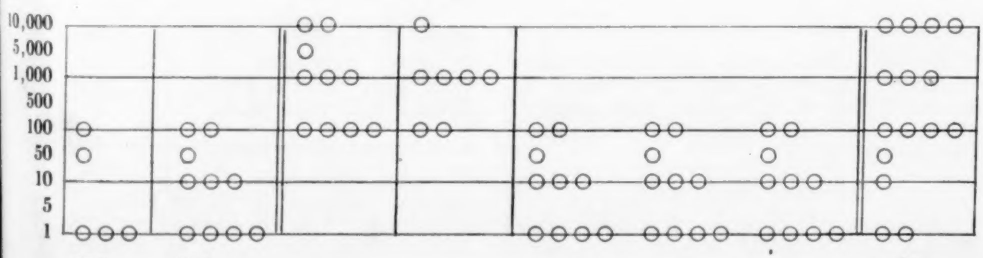
1285 and 431 make 1716

In addition, the addends are indicated on the counting board. Then whenever a line has five counters on it, as is the case with the tens line and the hundreds line in the case given here, the five counters are picked up and one is "carried" to the next space. In the example under consideration, there now are two counters in the fifties space. But two fifties make one hundred, so these two counters are picked up and a counter is laid on the hundreds line. The process is repeated until no line has more than four counters and no space more than one.

In subtraction, minuend and subtrahend are entered on the counting board. Then the counters of the subtrahend are matched with those of the minuend and each pair is removed from the board. In some cases it is necessary to "borrow" a counter of higher value from the minuend and to replace it by the equivalent value of counters in the next lower space or line. The process continues until no counters are left in the subtrahend.

To multiply a number by 10, the counters are laid out as if the tens line were actually the units line. To multiply by 100, the hundreds line represents the units. To multiply by 200, you multiply by 100 twice. Since 50 is half of 100, multiplying by 50 is accomplished by taking half of the number of counters on each line or space in 100 times the number. In the following example the number 284 is multiplied by 153.

(See solution below)



153 284 multiplied by 100, by 50, by 3 makes 43,462.

Division is difficult and a number of different methods are used. The computer is expected to know the multiplication facts. He decides on the proper quotient figure, and subtracts the partial product from the dividend, using the various lines as the units line as was done in multiplication.

Except for the process of division, computation with the loose counter abacus made no demands on the learner beyond learning how to enter the counters, how to read a number represented by counters, and how to count to five. Multiplication was clumsy. Division demanded a knowledge of multiplication combinations unless the computer avoided this issue by using repeated subtraction.

The loose counter abacus is simpler and slower than is the Chinese or the Japanese abacus which requires more mental work. On the other hand, it is a simpler device and one which is easier to master.

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BOOK SECTION

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This section presents the latest books which have been received for review in The Mathematics Teacher. Reviews of many of these books will appear in the monthly issues. Members of the Council are invited to send us further comments or corrections of errors relating to any of the books mentioned. In addition, a free loan service is being arranged whereby any member may borrow any of the books listed for a period not to exceed two weeks. Requests should be addressed to The Mathematics Teacher, 212 Lunt Building, Northwestern University, Evanston, Illinois.

BOOKS RECEIVED

High School

1. General Mathematics

Mathematics to Use, by Mary A. Potter, Supervisor of Mathematics, Racine, Wisconsin; Flora M. Dunn; Emmy Huebner Allen; John S. Goldthwaite. Cloth, ix+500 pages, 1950. Ginn and Company, Statler Building, Boston. $2.40.

2. Plane Geometry

Geometry, Meaning and Mastery, by Samuel Welkowitz, Harry Sitomer, and Daniel W. Snader. Cloth, v+512 pages, 1950. John C. Winston Company, 1006–1024 Arch Street, Philadelphia. $2.60.

3. Special Courses in Mathematics

Essentials of Business Arithmetic, Third Edition, by Edward M. Kanzer, Instructor of Business Education, Teachers College, Columbia University; and William L. Schaaf, Associate Professor of Education, Brooklyn College. Cloth, vii+476 pages, 1950. D. C. Heath and Company, 285 Columbus Avenue, Boston. $2.36.

College

1. Algebra

Elements of Algebra, by Lyman C. Peck, M.S., Department of Mathematics, Ohio State University. Cloth, xiii+230 pages, 1950. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York. $2.75.

2. Calculus and Analytic Geometry

Calculus and Analytic Geometry, by Cecil Thomas Holmes, Ph.D., Professor of Mathematics, Bowdoin College. Cloth, x+416 pages, 1950. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York. $4.75.

3. Statistics

First Course in Probability and Statistics, by J. Neyman, University of California. Cloth, ix+350 pages, 1950. Henry Holt and Company, 257 Fourth Avenue, New York. $3.50.

Statistical Decision Functions, by Abraham Wald, Professor of Mathematical Statistics, Columbia University. Cloth, ix+179 pages, 1950. John Wiley and Sons, Inc., 440 Fourth Avenue, New York. $5.00.

4. Advanced Mathematics

Carus Monograph No. 9. The Theory of Algebraic Numbers, by Harry Pollard, Cornell University. Cloth, xii+143 pages, 1950. The Mathematical Association of America, University of Buffalo, Buffalo. $3.00.

Carus Monograph No. 10. The Arithmetic Theory of Quadratic Forms, by Burton W. Jones,

University of Colorado. Cloth, x+212 pages, 1950. The Mathematical Association of America, University of Buffalo, Buffalo. $3.00.

Elements of Ordinary Differential Equations, by Michael Golomb, Associate Professor of Mathematics, Purdue University; and Merrill Shanks, Associate Professor of Mathematics and Aeronautical Engineering, Purdue University. Cloth, ix+356 pages, 1950. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York. $3.50.

Linear Integral Equations, by William Vernon Lovitt, Ph.D., Professor of Mathematics, Colorado College. Cloth, ix+250 pages, 1950. Dover Publications Inc., 1780 Broadway, New York. $3.50.

TEACHING OF MATHEMATICS

Hessische Beitrage zur Schulreform, by H. Wilh. Haupt, Direktor des Landesschulbeirats für Hessen. Paper, 24 pages, 1949. Publisher, Metopen Verlag Gmbh., Wiesbaden, Germany.

The Problem of Problem-Solving in Mathematical Instruction, by Aaron Bakst, School of Education, New York University. Paper, ii+53 pages, 1950. New York University Bookstore. $0.40.

MISCELLANEOUS

Arithmetical View Points: An Introduction of Mathematical Thinking, by Sidney G. Hacker, Professor of Mathematics, The State College of Washington. Paper vii+144 pages, 1948.

Evaluative Criteria, 1950 Edition, Cooperative Study of Secondary-School Standards. Cloth, 305 pages, 1950. Cooperative Study of Secondary-School Standards, Washington 6, D. C.

Strategy in Poker, Business and War, by John McDonald. Cloth, 128 pages, 1950. W. W. Norton and Company, Inc., 101 Poplar Street, Scranton 9, Pennsylvania. $2.50.

REVIEWS

Numbers We See. Anita Riess, Maurice Hartung and Catharine Mahoney. Teacher's Edition. Chicago, Scott, Foresman and Company, 1948. 162 pp., $1.32.

Numbers We See is a strikingly different textbook. It is made up completely of attractive, colorful pictures portraying activities of interest to children in the first grade. By means of these pictures, children are prepared for the development of the five following concepts: (1) counting, (2) basic facts, (3) measuring, (4) the number system, and (5) the use of money. There are no printed words throughout the seventy pages of textbook material, and written numbers are first introduced on page sixty-two.

The teacher's edition has additional pages (71–162) on which are instructions and suggestions for using the text. Each lesson has a definite objective, and is not something designed to merely keep the child amused or busy.

Frames, charts, and markers are available as teaching aids for some of the lessons. These increase the variety of pupil activity as well as increase the effectiveness of instruction.

The book does what the authors set out to do in providing a suitable textbook to be put into the hands of children before they have learned to read. The series of lessons gives the children a readiness for the further development of number concepts.—WINNIE MACON, Haskell Institute, Lawrence, Kan.

Plane Geometry. William G. Shute, William W. Shirk, and George F. Porter. New York, American Book Company, 1949. viii+406 pp., $2.48.

The authors of this text have not attempted to make the material deal with reasoning in everyday situations, but stress the type of reasoning necessary for proving theorems and solving problems related to these theorems, corollaries, and definitions. Geometric problem solving is applied in a very few cases to mechanics, navigation and the building trades. The teacher will find it necessary to supply most values of this type which are not covered adequately. The content will be of value to the teacher desiring a logical and sequential development of a subject based on many fundamental facts and principles. Many short exercises involve applications of definitions, theorems and corollaries that have been developed several pages before and require the student to identify those which are needed to solve the problems.

Nine full page photographs show applications of geometry occurring in such categories as industry, surveying and nature. Historical notes are frequently injected and serve to add interest and variation. Provisions for testing are frequent in the form of practice, achievement and review tests. These tests are of the true-false, multiple-choice and completion types.

A ten page appendix includes more involved material for possible use and considers such concepts as those involved in the principles of continuity, trigonometry and the golden section. Two hundred seventy-two exercises are included for the advanced students to work.—RODERICK C. MCLENNAN, Arlington Heights Township High School, Arlington Heights, Illinois.

Making Mathematics Work. Gilbert D. Nelson and Herschel E. Grime. Boston, Houghton-Mifflin Co., 1950., ix+630 pp., $2.40.

Making Mathematics Work is a textbook suitable for a ninth grade class in general mathematics. There is an abundance of photographs and cartoons of mathematical significance.

The first section deals with problem solving in general, and the first topic is analyzing problems. This topic, which is so very difficult for many children, is presented in an interesting and challenging manner.

The section on "Arithmetic Revisited" is a review of arithmetic introducing some new material which makes the review something besides mere drill.

"Earning and Managing Money" deals

largely with problems within the experience of most ninth-grade children. Investment, travel, insurance, and taxes are presented in a way to capture interest by including such diagrams as that showing the distance required to stop a car.

The sections on statistics, geometry, and algebra all emphasize the practical viewpoint.

The book includes thirty-six tables each of which is introduced when and where it is to be used.—WINNIE MACON, Haskell Institute, Lawrence, Kan.

Basic Mathematics for General Education. H. C. Trimble, F. C. Bolser, and T. L. Wade, Jr. New York, Prentice-Hall, Inc., 1950. xiii +313 pp., $3.25.

These professors from Florida State University may well be pleased with their attempt to build a better course in mathematics-for-general-education (required there), a course "aimed at the mathematical needs of some group of readers." Here, the group is college freshmen who lack the background for modern courses in the sciences.

The outstanding feature of the text is its its conversational style coupled with the unity which results from building around the theme, "mathematics is a language." The presentation is informal, unconventional, catchy, clear, and addressed directly to the reader. Such chapter titles as "Equations as Algebraic Sentences" and "The Language of Exponents" help show how the theme is carried forth.

In reading, one encounters discussions of types of reasoning, our number system, equations, ratio, approximate computation, exponents, variation, similar figures and indirect measurement, expressing relationships, business arithmetic, statistical data, the scientific method, and the citizen and mathematics.

Additional noteworthy features are frequent summaries, chapter introductions to acquaint the readers with what is to follow, exercises dispersed throughout the chapter, historical references, and the logical development of the material paralleling the student's growth.

The text has a good format. It contains a preface, table of contents, introduction, several diagrams, a bibliography, an appendix, tables, answers to the exercises, and a workable index.

The text would seem to have possibilities for use in junior colleges, vocational schools, and for persons having to study mathematics alone.

It has the definite advantage of having been used in its preliminary stage. Discoveries made here are discussed in the preface and should prove helpful to teachers using this text.

The authors certainly sell the need of "mathematics for all."—J.J.S.

The Anatomy of Mathematics. R. B. Kershner, and L. R. Wilcox. New York, The Ronald Press Company, 1950. xi+416 pages, $6.00.

The authors of this book set for themselves the task of describing the ultimate and intimate logical structure—the *anatomy*—of mathematics. Throughout the entire volume their emphasis is on mathematics as a mental discipline; their keynote is the axiomatic method. "The only prerequisites for reading this book are the desire to start and the perseverance to finish. The reader does not even need to know the sum of 7 and 5; incidentally, if he does not know this sum, he will not learn it from this book."

Early in the text the authors take up a discussion of language itself, and point out the need for careful definitions. A careful analysis of the process of acquiring *meanings* for words and the building-up of a language basis is followed by a discussion of the relation of logic and mathematics to that basis. The meaning of the word *mathematics* itself is discussed at great length, from its earlier definition as the science of number, the science of measurement, the science of space, down to the present definition as the science of axiomatics. The reader will be delighted with the fanciful history of the beginnings of mathematics as the science of number enacted by a hypothetical primitive tribal chieftain faced with the problem of deciding whether he should or should not engage his rival in battle.

The stage now being carefully set, the reader is unhurriedly introduced to the concepts of elements, sets, subsets, algebra of sets, and ordered pairs. Next he makes acquaintance with the ideas of relation and function, and is grounded in postulational method, implication, statements and proofs. The game of black and white hats is used to explain the nature of indirect proof. Upon the foregoing framework the authors now proceed to erect the elementary theory of groups, positive integers, and finite sets. This is followed by discussion of inductive definition and the principle of choice, extended operations, the fundamental theorem of arithmetic, and infinite sets. The theory of positive integers is rounded out with discussion of isomorphism, categorical systems of axioms, and equivalence and order relations. Next the positive rational number system is developed, followed by a discussion of one-dimensional continua, positive real numbers, real numbers, and general fields.

A number of projects designed to enable the reader to test himself appear throughout the book. Hints to solution and answers to these projects are contained in the appendix.

While it is expected that this book will be of service to high school or college teachers of mathematics or science, especially as a reference text, it is written for the reader with a meager technical background in mind. The development of the subject matter is very easy to follow, and the style of presentation is such as to retain reader interest to the very end.—E. W. BANHAGEL, Northwestern University, Evanston, Illinois.